Oct. 16

Team Round

General Rules and Answer Forms

You may use pencils, pens, scratch paper, graph paper, rulers, compasses, and protractors. However, calculators (also, slide rules and abbaci) may not be used.

Answer Forms

Fractions should be simplified.

For instance, 2/4 is incorrect and should be simplified to ½.

Fractions should be improper.

For instance, 1 ½ is incorrect and should be written as 3/2

Fractions should be used instead of mixed numbers, unless otherwise specified in the problem

For instance, 1.5 is incorrect and should be written as an improper fraction

Ratios should be reduced.

For instance, 2 : 4 (read "2 to 4") is incorrect and should be written 1 : 2.

Answers need to be exact unless otherwise specified.

For instance, 3.14 will not work for .

Radicals should be rationalized.

For instance, $\frac{1}{\sqrt{2}}$ is incorrect and should be written as $\frac{\sqrt{2}}{2}$.

Units are not necessary unless the answer is time, in which case A.M. or P.M. is required. However, if used, they must be correct.

Write your answers neatly in the space provided. Please make your handwriting legible. There is no guessing penalty. Both blank answers and incorrect answers will score 0 point. Finally, remember to write your name and team name on the answer sheet.

Any instructions included within a problem take precedence over the directions above.

Round Instructions

The Team round will consist of 16 problems, ranging from easy to hard, over 40 minutes. Students may work together with their teams for this round. The first 8 questions will be worth 10 points each and the second 8 questions worth 15 points each. No partial credit will be given unless specified otherwise.

When you are prompted to begin, you may start working on the questions. When you are prompted to stop, you should put down the pencil immediately. The proctor will pick up the answer sheets. The proctor will give 10 minutes and 1 minute warning.

Oct. 16

Problems

1. Evaluate $\left(5-\frac{20}{2}\right)\left(5-\frac{18}{2}\right)\left(5-\frac{16}{2}\right)\ldots\left(5-\frac{4}{2}\right)(5-\frac{2}{2})$.

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2. Blaire has a stack of books consisting of a red book, a green book, and yellow book, and a blue book. The red book is between the green and yellow books (but not necessarily right above or below either of them). The blue book is above both the yellow and red book. The yellow book is not at the bottom of the pile. List the colors of the four books in order from top to bottom.

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3. Tom bought some gerbils from a store and now he needs to buy cages for them. If he buys 3 cages, he will have one too many gerbils to put the same number into each cage. If he buys 7 cages, he will have one too few to put the same number of gerbils into each cage. What is the smallest number of gerbils that he could have?

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4. For what integer x does $x^{x/3}=256$?

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5. How many points (x,y) exist on the circle $x^2+y^2=169$ such that x and y are both integers?

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6. Andy, Bridgit, and Connor each have 40 pieces of candy. Andy gives half of his candy to Bridgit and Bridgit then gives half of her candy received to Connor. If Connor keeps one piece of candy and divides the rest equally among three dogs, how many pieces of candy does each dog receive?

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7. George starts at the origin on the Cartesian coordinate plane and walks halfway to the point (0, 8). Then, he turns and walks halfway to the point (8,0). He turns again and walks halfway to (0,8), and then turns and walks halfway to the point (8,0). How far is he now from his original spot?

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8. Out of the 5-digit binary strings consisting of the digits 0 and 1, how many have the property that, of the 3 substrings of three consecutive digits, one of them occurs at least twice? (For example, 1111 would have this property, but 10010 would not.)

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9. John is driving when he notices that his odometer reads 49894, a palindrome with three distinct digits! How many more miles will he have to drive before the odometer reads another palindrome with three distinct digits?

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10. Eva is taking a flight from New York to Chicago, and then a second one from Chicago to Seattle. Her first flight has a 20% chance of leaving late and her second flight has a 30% chance of leaving late. The only way she will fail to make it back to Seattle is if her first flight leaves late but her second does not. What is the probability she will be able to make it back to Seattle? Express your answer as a percent.

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11. Given that $a = \frac{1}{-2+\sqrt{5}}$ and $b = \frac{1}{2+\sqrt{5}}$, calculate $a^2 + b^2 + 7$.

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12. A sequence of integers $a_1, a_2, \ldots, a_{2010}$ satisfies the two following conditions:

- $a_2 = 1001$
- $a_{n+2} = a_{n+1} - a_n$ for $2008 \geq n \geq 1$

What is the value of $a_1 + a_2 + a_3 + \cdots + a_{2010}$?

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13. Four girls and four boys are sitting in a row of eight chairs. How many ways can they be seated so that no girls sit next to each other and no two boys sit next to each other?

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14. Let ABC be a triangle. The angle bisectors of B and C meet CA and AB at E and F, respectively. If $\angle BEF = 25°$ and $\angle CFE = 15°$, what is the measure of angle $\angle BAC$?

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15. Given that x and y are positive integers, $xy + x + y = 23$ and $x^2y + xy^2 = 120$, find the value of $x^2 + y^2$.

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16. P is a point inside triangle ABC. AP, BP, and CP meet BC, CA, and AB at D, E, and F respectively. If $\frac{AF}{FB} = \frac{3}{2}$, $\frac{BD}{DC} = \frac{1}{3}$, and $\frac{CE}{EA} = 2$, what is the value of $\frac{BP}{PE}$?