

Puzzle Round

General Rules and Answer Forms

You may use pencils, pens, scratch paper, graph paper, rulers, compasses, and protractors. However, calculators (also, slide rules and abacaci) may not be used.

Answer Forms

Fractions should be simplified.

For instance, $\frac{2}{4}$ is incorrect and should be simplified to $\frac{1}{2}$.

Fractions should be improper.

For instance, $1\frac{1}{2}$ is incorrect and should be written as $\frac{3}{2}$

Fractions should be used instead of mixed numbers, unless otherwise specified in the problem

For instance, 1.5 is incorrect and should be written as an improper fraction

Ratios should be reduced.

For instance, 2 : 4 (read “2 to 4”) is incorrect and should be written 1 : 2.

Answers need to be exact unless otherwise specified.

For instance, 3.14 will not work for π .

Radicals should be rationalized.

For instance, $\frac{1}{\sqrt{2}}$ is incorrect and should be written as $\frac{\sqrt{2}}{2}$.

Units are not necessary unless the answer is time, in which case A.M. or P.M. is required. However, if used, they must be correct.

Write your answers neatly in the space provided. Please make your handwriting legible. There is no guessing penalty. Both blank answers and incorrect answers will score 0 point. Finally, remember to write your name and team name on the answer sheet.

Any instructions included within a problem take precedence over the directions above.

Round Instructions

The Puzzle round will consist of 5 very difficult problems over 40 minutes. Students may work together with their teams for this round. Each question will be worth 20 points, for a maximum of 100 points. No partial credit will be given unless specified otherwise.

When you are prompted to begin, you may start working on the questions. When you are prompted to stop, you should put down the pencil immediately. The proctor will pick up the answer sheets. The proctor will give 10 minutes and 1 minute warning.

1. How many pairs of real numbers (x, y) satisfy the following equation?

$$x\sqrt{y} + y\sqrt{x} - \sqrt{2010x} - \sqrt{2010y} = \sqrt{2010xy} - 2010$$

2. A target is devised with 4 concentric circles of radii 1,2,3 drawn on it. George throws three darts of diameter 1 at the target. The first lands completely but randomly with the circle of radius 1, the second completely but randomly within the circle of radius 2, and the third completely but randomly within the circle of radius 3. What is the probability that none of the 3 darts overlap?

3. A rectangle $ABCD$ with $AB = 4$ and $BC = 2$ is folded such that C lands on top of A , forming a pentagon. This pentagon can be divided into three triangles such that each triangle has a vertex at A . Determine the ratio of the area of the middle triangle to the area of the entire pentagon.

4. Carl and David, who are starting a club together, each have 10 friends. Carl will forward an invitation to some number of his friends, and then David will forward invitations to some of his own friends such that they send a total of 10 invitations. If none of their friends overlap, how many possible groups of friends could they invite to their club?

5. Mary and Nancy are playing a numbers game. Nancy tells Mary five not necessarily distinct positive integers. Mary starts with a score of 0, and then, for each of Nancy's five numbers, either adds it to her score, subtracts it from her score, or ignores it.

(For example, if Nancy's five numbers were 1,2,3,4,5, possible scores for Nancy would include $1+2+3+4-5 = 5$, $1 = 1$, $1+5 = 6$, etc.)

Nancy wants to tell Mary five numbers such that Mary can use them to get as many positive consecutive prime scores, starting from 2, as possible. (For example, if Nancy was only giving Mary three numbers, which happened to be 2, 3, 4, Mary could get scores of 2, 3, 5, and 7.) Write the five numbers, separated by commas, that Nancy should tell Mary (the order that you write the numbers does not matter).

Your team will gain 2 points from this problem for each consecutive prime score greater than 7 that your set of numbers allows Mary to obtain, up to 20 points (even if your combination allows her to get more than 14 consecutive primes total). You will not receive a negative score for this problem.