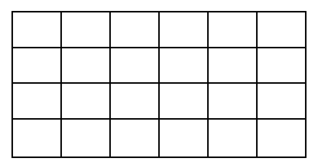
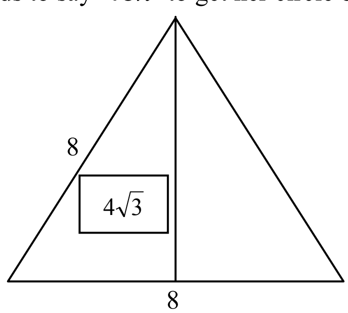
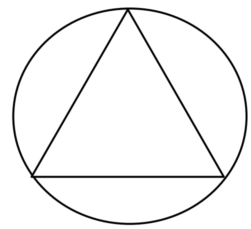
Rocket City Math League

Gemini (Geometry) Solutions

2008-2009
Round 1

<table>
<tr><td>1. Each cookie has a surface area of $6s^2$, where s is the side length. This means each cookie has a surface area of 24. Multiply 12 (one dozen) by 24 to get 288.</td><td>288</td></tr>
<tr><td>2. Since the point (5,-3) is on the circle, the distance from the point to the center (-2,4) will be the radius. Use the distance formula:
radius = $\sqrt{(x-h)^2+(y-k)^2}$ (x,y)=(5,-3) (h,k)=(-2,4)
$\sqrt{(5+2)^2+(-2-4)^2}$
$\sqrt{(7)^2+(-7)^2}$
$\sqrt{49+49}$
$\sqrt{98}$
$7\sqrt{2}$</td><td>$7\sqrt{2}$</td></tr>
<tr><td>3. Use Heron's formula to find the area of the triangle.
Area $=\sqrt{s(s-a)+(s-b)(s-c)}$ where *s* is the semi-perimeter of the triangle and *a, b,* and *c* are the sides
$S=\frac{1}{2}$ the total perimeter of triangle $=\frac{1}{2}(4+8+10)=11$; a=4, b=8, c=10
$A=\sqrt{11(11-4)(11-8)(11-10)}$
$=\sqrt{11(7)(3)(1)}$
$=\sqrt{231}$</td><td>$\sqrt{231}$</td></tr>
<tr><td>4. Consider a rectangle that is NxN units. A rectangle is defined by choosing two of the N+1 vertical lines and two of the N+1 horizontal lines (remember to count the edges of the rectangle) to get the 4 sides of a rectangle. So the way to find the number of rectangles is to find out how many ways you can choose two lines from each of the N+1 lines. This requires the combination formula.

The formula is ${}_nC_r=\frac{n!}{r!(n-r)!}$ where ! denotes factorial.
In this case, the numbers will be: ${}_{n+1}C_2=\frac{(n+1)!}{2!(n+1-2)!}=\frac{(n+1)!}{2!(n-1)!}=\frac{(n+1)(n)(n-1)(n-2)...}{2(n-1)(n-2)...}$
As you can see, everything but the first two terms in the numerator and the first term in the denominator will cancel, leaving $\frac{(n+1)(n)}{2}$, which you can use for both length and width. Plug in the given length and width and multiply the two answers together. $\frac{(6+1)(6)}{2}\times\frac{(4+1)(4)}{2}=\frac{42}{2}\times\frac{20}{2}=21\times10=210$</td><td>210</td></tr>
<tr><td>5. The standard equation of a circle is $(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center of the circle, and r is the radius. First divide the equation by 2 to put in standard form, making the equation $x^2+y^2=16$. In this case $r^2=16$, so radius=4. The area of the circle is $\pi(r)^2=\pi(4)^2=16\pi$. Now, if the farmer gets 5 cobs for every 2π square units, and the area of the circle made by the space ship was 16π square units, divide the area of the circle by 2π to find the number of square units and multiply that by 5 to find the number of cobs of corn lost.
$(\frac{16\pi}{2\pi})(5)=40$ cobs</td><td>40</td></tr>
<tr><td>6. Since the sphere is inscribed in the cube, the diameter of the sphere is equal to the side of the cube, 6. The volume of the sphere is $\frac{4}{3}(\pi)r^3=\frac{4}{3}(\pi)(3)^3=36\pi$. The volume of the cube is $6^3=216$. Therefore, the volume outside the sphere but inside the cube is the difference between the volume of the cube and the volume of the sphere, or $216-36\pi$</td><td>$216-36\pi$</td></tr>
</table>

7. apothem= $\frac{s\sqrt{3}}{2}$, where s equals side of the polygon $6 = \frac{s\sqrt{3}}{2}$ $12 = s\sqrt{3}$ $s = 4\sqrt{3}$ $Area = (6)\frac{s^2\sqrt{3}}{4} = (6)\frac{(4\sqrt{3})^2\sqrt{3}}{4} = (6)\frac{(48)\sqrt{3}}{4} = (6)(12\sqrt{3}) = 72\sqrt{3}$	$72\sqrt{3}$
8. Since the lines are parallel, find the distance between the two lines by taking one point on one of the lines and finding the distance from that point to the other line. Find a point on one of the lines. (1, -1) satisfies the equation 3x=y+4. Now we have the point (1, -1) and the line 3x-y-6=0. The formula for the distance from a point to a line is $\frac{\lvert Ax+By+C\rvert}{\sqrt{A^2+B^2}}$ where $Ax+By+C=0$ is the equation of the line, and x and y are the coordinates of the point. Plugging in the given information, we get $\frac{(3)(1)+(-1)(-1)+(-6)}{\sqrt{(3)^2+(-1)^2}} = \frac{\lvert 3+1-6\rvert}{\sqrt{10}} = \frac{2\sqrt{10}}{10} = \frac{\sqrt{10}}{5}$	$\frac{\sqrt{10}}{5}$
9. The basic question is: a circle is inscribed in a hexagon, what is the area of the circle? A quick rule is that the radius of the circle inscribed in a polygon is area of polygon divided by semi-perimeter. To find the area imagine the hexagon as six triangles. These become the lengths of the side of one of the triangles. Using the formula $\frac{1}{2} base \times height$, the area of the triangle is $16\sqrt{3}$. Since there are six of these triangles, the total area of the hexagon is $96\sqrt{3}$. The semi perimeter, half of the perimeter, is 24. Plugging this into our equation, we get $\frac{96\sqrt{3}}{24}$, or $4\sqrt{3}$ as the radius of the circle. Thus, Nala needs to say 48π to get her circle back. 	48π
10. $SA = 6s^2 = 162$ $s^2 = 27$ $s = 3\sqrt{3}$ $diagonal = s\sqrt{3} = (3\sqrt{3})(\sqrt{3}) = 9$	

11. To find the height of triangle GHS, use Pythagoras's theorem. $12^2 - 6^2 = h^2$ $144-36=h^2$ $h=6\sqrt{3}$ Area of triangle GHS= $\frac{1}{2}bh = \frac{1}{2}(12)(6\sqrt{3})=36\sqrt{3}$ Apothem= $\frac{1}{3}h$ Apothem= $2\sqrt{3}$ Radius= $2\sqrt{3}$ Height= $\frac{3}{2}$ radius= $3\sqrt{3}$ Area of triangle MAO= $\frac{1}{2}bh$ $A=\frac{1}{2}(6)(3\sqrt{3})=9\sqrt{3}$ $\frac{9\sqrt{3}}{36\sqrt{3}}=\frac{1}{4}$ 	$\frac{1}{4}$
12. This is an infinite geometric series. The total distance traveled will be the distance the ball tavels going down (S_1) plus the distance the ball travels going up (S_2). Let $S_1 = 63 + \frac{2}{3}(63) + \frac{2}{3}\cdot\frac{2}{3}\cdot 63 + \ldots$ $\frac{2}{3}(S_1) = 44 + \frac{2}{3}(63) + \frac{2}{3}\cdot\frac{2}{3}\cdot 63 + \ldots$ Subtracting these two equations: $S_1 - \left(\frac{2}{3}S_1\right) = 63 + \frac{2}{3}(63) + \frac{2}{3}\cdot\frac{2}{3}\cdot 63 + \ldots$ $- \left(\frac{2}{3}(63) + \frac{2}{3}\cdot\frac{2}{3}\cdot 63 + \ldots\right)$ $\frac{1}{3}(S_1) = 63$ $(S_1) = (3)(63) = 189$ Alternatively, $(S_1) = \frac{a_1}{1-r} = \frac{63}{1-\frac{2}{3}} = \frac{63}{1/3} = 189$ Because every down bounce, with the exception of the initial 63 feet traveled from original height, must have an equal up bounce: $(S_2) = S_1 - 63 = 189 - 63 = 126$ Thus, the total distance traveled = $S_1 + S_2 = 189 + 126 = 315$ feet	315