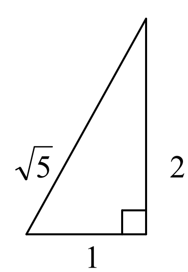
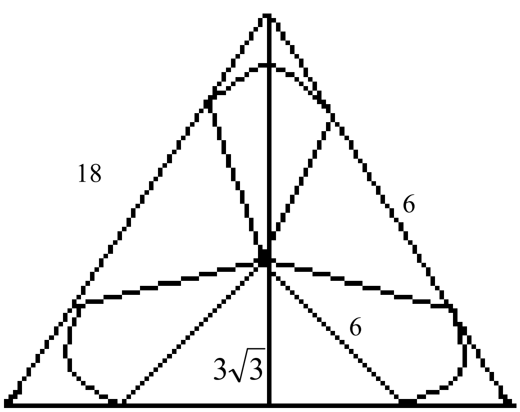
Rocket City Math League

Discovery Round 2 Solutions

2008-2009
Round 3

1. $3^3 = 27$ and $2^2 = 4$ $27 - 4 = 23$	**23**
2. $2^{14} = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2 = 2^{10}\times2^4 = 1024\times16 = 16384$	**16384**
3. Since $A \cup B = \{1,2,3,4,...,100\}$, $\{1,2,3,4,...,100\}$ is the set of all numbers that are in either A or B, because this is how the union, $\cup$, of two sets is defined. However, since $A \cap B = \{50\}$, 50 is an element of both A and B and thus should not be included. The sum of all the numbers fro 1 to 100 except 50 is the sum from 1 to 100, minus 50. The sum of 1 to 100 = $\frac{(1+100)}{2}\times100 = 5050$ and $5050 - 50 = 5000$	**5000**
4. $\frac{11x-2}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$ $\frac{11x-2}{(x+3)(x+2)} = \frac{A(x-2)+B(x+3)}{(x+3)(x-2)}$ $11x - 2 = Ax - 2A + Bx + 3B$ $11x - 2 = (A+B)x + (3B - 2A)$ From this we can conclude that $A + B = 11$, since the sum of the x terms on the left must equal the sum of the x terms on the right.	**11**
5. Dividing these two equations, we get $\frac{a\sin b}{a\cos b} = \frac{4}{2}$ $\tan b = 2$ $b = \tan^{-1}(2)$ b could also be equal to π more than this. However, if this were true, then sinb would be negative, meaning that a must be negative for $a\sin b = 4$ to be true. A negative value for a would not be the maximum value of a, so it can be disregarded. Then, plugging this in for b into one of the original equations, we get $a\sin(\tan^{-1}(2)) = 4$ Using the right triangle shown we find that $\sin(\tan^{-1}(2)) = \frac{2}{\sqrt{5}}$, so $a(\frac{2}{\sqrt{5}}) = 4$ $a = 2\sqrt{5}$  	$2\sqrt{5}$
6. For Lihkin, $t = x^2 - 1$ and $y = t + 7$ so $y = (x^2 - 1) + 7 = x^2 + 6$. For Werdna, $t = x + 2$ and $y = t^2 - 4$ so $y = (x+2)^2 - 4 = x^2 + 4x$ Set the two equations equal and $x^2 + 6 = x^2 + 4x$, so $4x = 6$ and $x = \frac{3}{2}$	$\frac{3}{2}$
7. ((3!)!)!=(6!)!=720! The number of zeroes at the end of 720! is equal to the number of 5's in 720 because there are more 5's than 2's in the prime factorization of 720!. The multiples of 25 will make 2 zeroes, but will only be counted once so the number of 25's must be found. The same logic can be used for 125 and 625. Therefore the number of zeroes at the end of 720! is equal to 720 divided by 5 plus 720 divided by 25 plus 720 divided by 125 plus 720 divided by 625.	**178**

$\frac{720}{5}=144$, $\frac{720}{25}=\frac{144}{5}=28R4$, $\frac{28}{5}=5R3$, $\frac{5}{5}=1$ $144+28+5+1=178$	
8. The greatest possible value of and ABC occurs when A, B, and C are consecutive and the angle is an internal angle of the decagon. The formula for the internal angle in degrees of any n-gon is $\frac{180\times(n-2)}{n}$ so for the degree measure of an internal angle of a decagon, it is $\frac{180\times(10-2)}{10}=144$	**144**
9. The number of way to choose y objects from a group of x objects when y is greater than x is 0. X choose 0 is 1 and x choose 1 is x. A pattern forms of 0,1x,0,1,x… and ends with 0	**0**
10. $20!=20\times19\times18\times17\times16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1$ $=19\times17\times13\times11\times7^2\times3^8\times2^{14}\times10^4$ The ten to the 4th can be ignored because we are trying to find the first digit left of the zeroes. Since the units digit of the problem excluding the tens, can only be influenced by the units digits of the numbers left, then the units digit is equal to the units digit of $9\times7\times3\times1\times9\times1\times4=4$	**4**
11. Using *'s to represent balloons and |'s to divide the balloons into 5 portions where the number of *'s in a portion represents the number of balloons that that alien child gets. For instance **|***|*|**** would be the first child with 2, the second child with 3, the third child with 1 and the last child with 4. ||||********** would be the first four kids with 0 balloons and the fifth child with all 10. The number of ways to arrange these items is equal to the number of ways to choose 4 dividers (|) from 14 items, or ${}_{14}C_4=\frac{14!}{(4!)(10!)}=1001$	1001
12.  A point equidistant from the vertices of an equilateral triangle is in a ratio of 2:1 of the height above to the height below, as seen in picture above. Using the side length 18 we find the height of the big triangle to be $9\sqrt{3}$ and using the 2:1 ratio we find the height of the smaller triangle to be $3\sqrt{3}$ then using the numbers 6 and $3\sqrt{3}$ we find that the smaller triangle is an equilateral triangle with side length 6. So, the total area of the triangles is $3\left(\frac{6^2\sqrt{3}}{4}\right)=27\sqrt{3}$. The partial circles combine to half of a circle because the triangles all touch the circle and their angles total to $180°$ therefore, half the circle. So the total area of the partial circles is $\frac{1}{2}\times6^2\times\pi=18\pi$ which makes the total area $18\pi+27\sqrt{3}$	$18\pi+27\sqrt{3}$