



Rocket City Math League

Discovery Test

2007-2008
Round 2

Answers must be written inside the adjacent answer boxes. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as improper fractions (unless otherwise specified in the problem). **No calculators, books, or other aids may be used.**

1. In order to save the Earth from a giant asteroid, NASA must know the diameter of the approaching asteroid. The diameter is equal to the sum of the solutions of the equation $x^2 - 68x + 256 = 0$. Find the asteroid's diameter. (1 point)	
2. Emperor Zargot can completely destroy a galaxy in 6 days. Emperor Capot can completely destroy the same galaxy in 5 days. Lord Destructo can completely destroy the same galaxy in 4 days. How many days would it take them to completely destroy the galaxy if they were working together? (1 point)	
3. What is the measure of the smaller angle formed by the minute hand and the hour hand of a clock at 4:54 (in degrees)? (1 point)	
4. The Evil Dr. McMeen is building a bomb large enough to destroy the planet Itsareallybigplanet. In order to build the bomb, however, the Evil Dr. McMeen must know how many distinct solutions exist to the equation $\cos^2 \theta - \sin^2 \theta = \frac{1}{2}$ on the interval $[0, 2\pi]$? Find the number of solutions to the equation on this interval. (1 point)	
5. If 5 cards are randomly dealt from a standard 52-card deck, what is the probability that these five cards contain four 2's? (2 points)	
6. Michael the Milky Way Magician is on his way to a magic show where he will use his magic wand to completely simplify the trigonometric expression $(\tan 10^\circ)(\tan 20^\circ)(\tan 30^\circ)(\tan 40^\circ)(\tan 50^\circ)(\tan 60^\circ)(\tan 70^\circ)(\tan 80^\circ)$. What is the simplest form of Michael's expression? (2 points)	
7. The sum of the following infinite series $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots$ is equal to the amount of fuel in tons that the Spaceship Supernova needs to travel from Earth to the star Orion. Determine the amount of fuel in tons that Supernova will need in order to reach Orion. (2 points)	
8. Nikhil's numeric eccentricity is equal to the eccentricity of his famous "Equation of the Day." If Nikhil's equation of the day is $16x^2 - 128x + 9y^2 + 126y + 553 = 0$, find the value of Nikhil's numeric eccentricity. (2 points)	
9. The space value of positive number, S, is defined to be the absolute value of the difference between the sum of the integers from 1 to S and the product of the integers from 1 to S. What is the largest space value less than 1000? (3 points)	
10. License plate numbers on the planet Neptune consist of 5 distinct digits that are always arranged in increasing order from left to right. If the leftmost digit on the license plate number cannot be 0, how many distinct license plate numbers are possible? (Neptune uses base 10 numbers for their license plates) (3 points)	
11. The centers of the faces of a regular tetrahedron are connected to form a smaller tetrahedron inside the original one. Find the ratio of the volume of the smaller tetrahedron to the volume of the larger tetrahedron. (3 points)	
12. Professor Quitoz loves the number 653. He is so obsessed with the number, that all of the problems on his math tests contain only the number 653. One day his students at the Martian Academy of Math and Science played a trick on him. They changed one of his questions so that it had a 17 in it. Professor Quitoz was devastated because he would have to rework the problem to find out the new correct answer. Help out the professor by finding the remainder when 653^{653} is divided by 17. (4 points)	

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