



## Rocket City Math League

### Mercury Test Solutions

**2007-2008**  
**Round 2**

Answers must be written inside the adjacent answer boxes. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as improper fractions. **No calculators, books, or other aides may be used.**

<p>1. The equation of a line perpendicular to another line has a slope that is the negative reciprocal of the original slope. So the equation of the line perpendicular to the line <math>4x - 7y = 22</math> has a slope of <math>-\frac{7}{4}</math>. By plugging in the point <math>(4, 8)</math>, the equation of the line in slope intercept form is <math>y = -\frac{7}{4}x + 15</math>.</p>	$y = -\frac{7}{4}x + 15$																																				
<p>2. 8 slices of pizza are in a box, so <math>\frac{7}{8} \times 8 + \frac{3}{8} \times 8 + \frac{3}{4} \times 8 + \frac{1}{2} \times 8 = 20</math>.</p>	<b>20</b>																																				
<p>3. <math>x</math>: original price of the spaceship  <math>.15x</math>: discount off ship  <math>x - .15x = .85x</math>: the price of the discounted ship            To find the price of the discounted ship, subtract <math>2100 - 1632.50 = 467.50</math>            Therefore <math>.85x = 467.50</math> or <math>x = \\$550</math></p>	<b>550</b>																																				
<p>4. The sum of Bryan's scores must equal 400 in order for Bryan to have exactly an average of 80 after five tests. To find the score needed on the last test, subtract the scores received from the needed sum of 400. <math>400 - 74 - 82 - 77 - 78 = 89</math></p>	<b>89</b>																																				
<p>5. The sum between the first two numbers is two, between the second and the third is five. Notice that we added three to the number that we added to get the next term. Therefore, from the third to the fourth, we add eight, between the fourth and the fifth, we add eleven, then fourteen between the next two. Therefore, the last two numbers are 57 and 77.  <math>57 + 77 = 134</math></p>	<b>134</b>																																				
<p>6. <math>N = 9</math> (1, 36, 2, 18, 3, 12, 4, 9, 6); <math>A = 72</math> hours (<math>24 \times 3</math>); <math>S = 2 + 3 + 5 + 7 + 11</math>, or 28  <math>9 + 72 + 28 + 72 = 181</math></p>	<b>181</b>																																				
<p>7. <math>x =</math> gallons of water  <math>\frac{4}{16+x} = 20\% = \frac{1}{5}</math>    <math>20 = 16 + x</math>, so <math>x = 4</math>.</p>	<b>4</b>																																				
<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 15%;"></th> <th style="width: 15%;">If the Criminal is</th> <th colspan="4" style="width: 70%;">Then the statements of</th> </tr> <tr> <th></th> <th></th> <th>Dave are</th> <th>PJ are</th> <th>Bryan are</th> <th>Patrick are</th> </tr> </thead> <tbody> <tr> <td></td> <td>Dave</td> <td>False</td> <td>True</td> <td>False</td> <td>True</td> </tr> <tr> <td></td> <td>PJ</td> <td>False</td> <td>False</td> <td>False</td> <td>True</td> </tr> <tr> <td></td> <td>Bryan</td> <td>True</td> <td>True</td> <td>False</td> <td>True</td> </tr> <tr> <td></td> <td>Patrick</td> <td>False</td> <td>True</td> <td>True</td> <td>False</td> </tr> </tbody> </table> <p style="margin-top: 10px;">The problem states that there is exactly one true statement out of the four given. If Dave is the criminal, there are two true statements (the Patrick's and Bryan's). If Bryan is the criminal, there are three true statements (all but his own). If Patrick is the criminal, there are two true statements (PJ's and Bryan's). However if the PJ is the criminal, there is only one true statement (Patrick). Therefore, only Bryan satisfies the conditions specified in the problem.</p>		If the Criminal is	Then the statements of						Dave are	PJ are	Bryan are	Patrick are		Dave	False	True	False	True		PJ	False	False	False	True		Bryan	True	True	False	True		Patrick	False	True	True	False	<b>Bryan</b>
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<p>9. The probability of a red alien receiving the blue ribbon is <math>\frac{7}{19}</math>. The probability of a green alien receiving the gold medal is <math>\frac{4}{18}</math>, assuming that the one red alien cannot win both awards. Therefore, the probability of both events occurring is <math>\frac{7}{19} \times \frac{4}{18} = \frac{14}{171}</math>.</p>	$\frac{14}{171}$
<p>10. Ekaj: <math>\frac{42}{60} = .7</math> mi/min      Llij: <math>\frac{54}{60} = .9</math> mi/min  <math>.7(x + 5) = .9(x - 5)</math>  <math>.7x + .35 = .9x - .45</math>  <math>0.80 = 0.2x</math>  <math>x = 40</math> minutes  <math>.7(40 + 5) = 31.5</math>  31.5 miles or <math>\frac{63}{2}</math> miles</p>	<b>31.5 or <math>\frac{63}{2}</math></b>
<p>11. <math>11^1 = 11</math>  <math>11^2 = 121</math> What is important to notice is that the ten's digit of any <math>11^x</math> term is equal to  <math>11^3 = 1331</math>  the units digit of <math>x</math> and the units digit is equal to 1. Therefore, if you subtract one from <math>11^{20}</math>, the tens digit is still zero.</p>	<b>0</b>
<p>12. If the company has a 1:2 odds of producing a spaceship with a defect, then it has a <math>\frac{1}{3}</math> probability of making a spaceship incorrectly. If a company has a <math>\frac{1}{3}</math> probability of producing a spaceship incorrectly, then the probability of producing a spaceship without a defect is <math>\frac{2}{3}</math>. Thus the probability of producing all six spaceships correctly is  <math>\left(\frac{2}{3}\right)^6 = \frac{64}{729}</math>. This means that the probability of one or more of the spaceships having a defect is <math>\frac{665}{729}</math>.</p>	$\frac{665}{729}$

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