Name_____ Math Level_____

Rocket City Math League

Gemini Test Solutions

2007-2008
Round 2

Answers must be written inside the adjacent answer boxes. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as improper fractions. **No calculators, books, or other aides may be used.**

1. $a^2 + b^2 = c^2$, so $\sqrt{9+36} = 3\sqrt{5}$.	$3\sqrt{5}$
2. Every rotation of the smaller circle travels a distance of 2.6 cm on the bigger circle. Since the bigger circle has a circumference of 7.8, divide the distance around the big circle by distance traveled by the smaller circle to determine how many rotations the smaller circle will make. $\frac{7.8}{2.6} = 3$	**3**
3. Since $x^2 + y^2 = r^2$ is the standard equation for a circle with radius *r* and the given circle is defined by the equation $x^2 + y^2 = 15$, the radius of the circle is $\sqrt{15}$. To find the area of the circle, $\pi r^2 = 15\pi$	15π
4. Since $(2,1)$ and $(8,1)$ lie on the same horizontal line, the distance between A and C is 6. The height from $(4,8)$ to the horizontal line created from A and B is 7. Therefore, the area is $\frac{1}{2}(6)(7) = 21$.	**21**
5. Because the dilation is applied 8 times, $x^8 = 6561$. Therefore, $x = 3$.	**3**
6. The measure of a side of the cube is $\frac{5}{\sqrt{3}}$. The surface area of one side of the cube equals s^2, so the surface area equals $\frac{25}{3}$.	$\frac{25}{3}$
7. Using the Pythagorean Theorem with the separation distance and the length of each rope, we find that the walls are 20 ft. and 36 ft. tall. Using the common formula $\frac{xy}{x+y}$ for the intersection of two ropes and substituting the given values, we find that the intersection point is $\frac{90}{7}$ feet above the ground.	$\frac{90}{7}$
8. The volume of a cone is $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(16)^2(h) = 2560\pi$, so $h = 30$. The radius, height, and slant height of the cone form a right triangle. So, $16^2 + 30^2 = s^2$, and the slant height, *s*, is 34.	**34**
9. To find the surface area of a hemisphere, use the formula $\frac{1}{2} \cdot 4\pi r^2$. The radii of the two hemispheres are 12 and 13. Plugging these values into the formula, we find that the surface areas are 288π and 338π respectively. Now, to find the cost of the rim, draw two concentric circles with radii 13 and 12. Subtract the area of the large circle from the area of the smaller circle: $169\pi - 144\pi = 25\pi$. $288\pi + 338\pi + 25\pi = 651\pi$, so $651\pi \times \$5 = \$3255\pi$	**3255 π**
10. The common radius of the hemisphere, cone, and cylinder is 6. The volume of the hemisphere = $\frac{1}{2} \cdot \frac{4}{3}\pi r^3 h = 144\pi$. The volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(17) = 204\pi$. The volume of the cylinder $\pi r^2 h = \pi(36)(23) = 828\pi$. Therefore, the total volume of the toy is $144\pi + 204\pi + 828\pi = 1176\pi$.	**1176 π**

11. Since the slopes of $\overline{AD}$ and $\overline{BC}$ are both -1, and the slopes of the other two sides are not equal, the figure is a trapezoid. Use the distance formula to determine the length of the base and the height of the trapezoid, and then use the perpendicular distance from the base to the height to determine the numeric value of the height. Doing so, AD = $8\sqrt{2}$, BC = $\sqrt{2}$, and the height is $\frac{5}{2}\sqrt{2}$. Using A = $\frac{1}{2}h(a+b)$, we get $A=\frac{1}{2}\cdot\frac{5\sqrt{2}}{2}\left(8\sqrt{2}+\sqrt{2}\right)=\frac{45}{2}.$	$\frac{45}{2}$
12. Using the outside times whole thing equals outside times whole thing, we get $a(2a-2)=x(6x)$, and $x(6x)=(x+1)(2x+4)$. The second equation gives $2x^2-3x-2=0$, so $x=-\frac{1}{2}$ and 2. Since length can only be positive, only $x=2$ works. Substituting this value in the first equation gives us $a^2-a-12=0$, so $a=4$.	**4**