



Rocket City Math League Discovery Test

**2006-2007
Round 3**

Answers must be written inside the adjacent answer boxes. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as common fractions (unless otherwise specified in the problem). **No calculators, books, or other aids may be used.**

1. Evaluate $(\log_5 4 + \log_{125} 16)(\log_2 5)$. (1 point)	
2. Find the determinant of the matrix $\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$. (1 point)	
3. If $f(x) = 2x + 1$ and $g(x) = \frac{1}{2}x - \frac{1}{2}$, what is the value of $f(g(5))$? (1 point)	
4. Evaluate $\sin 15^\circ + \cos 15^\circ$ and express your answer in simplest form. (1 point)	
5. Find the sum of the imaginary roots of $x^4 + 3x^3 + 3x^2 + 3x + 2 = 0$. (2 points)	
6. What is the sum of the areas of the four different closed figures that can be created by joining the endpoints of three congruent 60° arcs cut from a circle with area 8π ? (2 points)	
7. How many possible combinations exist for a, b, c, d, and e if $a+b+c+d+e=16$ and a, b, c, d, and e are all positive integers? (2 points)	
8. If the points of intersection of the graph with parametric equations $\begin{cases} x = 3 \cos t \\ y = 4 \sin t \end{cases}$ and the graph with polar equation $ \theta = \frac{\pi}{4}$ are the vertices of a closed, convex, polygon, find the area of that polygon. (2 points)	
9. A school's parking lot has parking spaces numbered sequentially from 1 to 9999. As a senior prank, a group of seniors erased the numbers that were divisible by 2, 3, or 7. How many numbers were not erased? (3 points)	
10. On rectangle RCML, \overline{RL} is 16 units long and \overline{LM} is 8 units long. Point P is chosen on \overline{RL} and the intersection of diagonal \overline{RM} and line segment \overline{PC} is point X. If the area of triangle PLM is twice the area of PMX, what is the area of triangle PRX? (3 points)	
11. Bobby selects four distinct integers between 1 and 50 inclusive, and x is the product of these four numbers. Bob uses the same process, and y is the product of the four numbers he selects. If $\log_{14} x$ and $\log_{14} y$ are both integers, what is the probability that Bobby and Bob chose exactly the same integers (irrespective of order)? (3 points)	
12. Quarter circle O has perpendicular radii \overline{OA} and \overline{OB} with length r . Quadrilateral OPQR is drawn such that point P is on \overline{OA} , point Q is on arc AB, point R is on \overline{OB} , and \overline{PQ} is parallel to \overline{OB} . Also, $\angle QRO$ is an acute angle. Given that \overline{PQ} has length 7 and \overline{QR} has length 6, what is the smallest possible area of the quarter circle? (4 points)	

The material on this page is the property of the Rocket City Math League. Reproduction other than for non-profit educational purposes is strictly prohibited without the expressed written consent of the RCML.