



# Rocket City Math League

## Discovery Test

**2006-2007**  
**Round 2**

Answers must be written inside the adjacent answer boxes. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as common fractions (unless otherwise specified in the problem). **No calculators, books, or other aids may be used.**

1. Simplify $(2 - 3i)^2 - (4 - 5i)$ where $i = \sqrt{-1}$ . (1 point)	
2. If $x \heartsuit y = \frac{98x - 98y}{y^3 - x^3}$ , then what is the value of $5 \heartsuit 3$ ? (1 point)	
3. A digital clock displays the time using a number of horizontal and vertical line segments (i.e. the digit 1 has 2 line segments and the digit 7 has 3 line segments, as shown in the diagram). Using exactly 20 line segments that can be arranged vertically or horizontally, what is the latest time that can be displayed? Assume a 12-hour clock is being used and 01:00 is the earliest time and 12:59 is the latest time. Write your answer in hour:minute format (i.e. 9:00). (1 point)	
4. Xebon and Yenmar have different coplanar circular orbits centered at the Sun, and both are traveling clockwise. Currently, Xebon is located between the Sun and Yenmar, and all three are collinear. If Xebon makes one revolution around the sun in 24 hours, and Yenmar makes one revolution around the sun in 72 hours, how many hours will elapse until the next time the three celestial objects are collinear, with Xebon between Yenmar and the Sun? (1 point)	
5. Farmer Bob owns a rectangular field, FARM, with FA and FM equal to 30 miles and 10 miles respectively. Bob would like to build a house at a single point along FM, the bank of a river. Bob owns a silo that is located five miles from M along MR, and a chicken coop is located at point A. Every morning, Bob must travel from his house to the chicken coop, back home, to the silo, and back home again. In order to minimize the distance of his daily trip, how far along FM from M should Bob build his house? (All structures are points and have negligible areas.) (2 points)	
6. Eric is taking a vocabulary test and has a word bank of four words. He uses each word once on a 4 problem fill-in-the-blank test and guesses randomly. The teacher tells him he answered at least one of the 4 questions correctly. What is Eric's expected score (expected value of his score) on this test if each question is worth one point for a total score of four points? (2 points)	
7. Elina, Dave, and Steve all want to pick some Bignogs from the Gabdoom tree. However, they see that there are only 25 Bignogs left. Being civilized members of society, the three decide to split the Bignogs up by giving Elina more than either Dave or Steve. If each takes at least one, then in how many ways can the three split the Bignogs up? (2 points)	
8. Four non-adjacent corners of a cube with side length 12 are removed. For each corner being truncated, a cut is made such that the cut passes through the three vertices that share an edge with the corner along the face diagonals of the cube. For example, one such cut is shown in the diagram to the right. What is the volume of the resulting solid after all four corners have been removed? (2 points)	
9. Evaluate: $\sum_{x=2}^{\infty} \frac{1}{x^3 - x}$ . (3 points)	
10. Maglorps the alien claims to have the fastest ship in the universe. However, to prove it, he must fly the famous Zazzob course. His flight takes exactly as long as the record, but he encountered some problems along the way. When he was halfway through the course relative to the total amount of time it took him to complete the course, two of Maglorps's engines malfunctioned. He was stopped for 10 seconds, and he finished the course at a third of his original speed. Later, while daydreaming, Maglorps realizes that if only one of his engines had malfunctioned and if the malfunction had occurred 10 seconds later than it actually did during his flight, stopping him for 10 seconds, and then slowing his speed by one-third of his original speed, he would have finished 40 seconds earlier than the record. How long did Maglorps take to fly the Zazzob course, in seconds, during his actual flight? (3 points)	
11. Find the sum of the solutions of $\frac{2}{\log_{2 \sin x} 3 - \log_{2 \sin x} 4} + \frac{1}{\log_{\cos^2 x} 3 - \log_{\cos x} 2} = 1$ in the interval $[-\pi, 2\pi]$ . (3 points)	
12. If x, y, and z are real numbers that satisfy, $x + \frac{1}{z} = 15$ , $y + \frac{1}{x} = 10$ , and $z + \frac{1}{y} = 5$ , what is the sum of the possible values of $xyz$ ? (4 points)	

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