



Rocket City Math League Discovery Test

**2006-2007
Round 1**

Answers must be written inside the adjacent answer boxes. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as improper fractions (unless otherwise specified in the problem). **No calculators, books, or other aids may be used.**

1. Last year, we determined that Steve's favorite number was one. This year, however, his new favorite number is equal to the distance from the origin to the point (6, 12). What is Steve's new favorite number? (1 point)	
2. Pick a positive integer. Add the year you were born. Then add the square of your age as of January 1, 2007. Add the number formed by last four digits of your home phone number. If the additive inverse of your result is A , and the multiplicative inverse of your result is B , then find $A \times B$. (1 point)	
3. What is the smallest positive value of n such that the expansion of $(2006 + 2006i\sqrt{3})^n$ has no imaginary term? Assume $i = \sqrt{-1}$. (1 point)	
4. Find the sum of all distinct values of x that satisfy the equation $(x^2 + 5x + 5)^{(x^2 + x - 6)} = 1$. (1 point)	
5. Evaluate the infinite expression $\frac{6}{1 + \frac{6}{1 + \frac{6}{1 + \frac{6}{\dots}}}}$. (2 points)	
6. Find the sum of the two least positive integers, x , such that $x^{2006} + x^{2007}$ is a perfect square. (2 points)	
7. What is the largest value of θ in the open interval $(0, \pi)$ that satisfies the equation: $\sin \theta - 2 \sin^3 \theta = -\sqrt{3} \cos \theta \cos 2\theta$? (2 points)	
8. What is the domain of $f(x) = \log(2x - 3) + \log(7 - x)$? (Give your answer in interval notation.) (2 points)	
9. Being bored during his 25 year voyage to Alpha Centauri, Space Captain Bonaparte begins playing with his arrow keys. He chooses one of the four keys (up, down, left, and right), and the cursor on the unbounded computer monitor moves one space in the direction chosen. In how many ways can he make 8 moves, so that the cursor finishes in its original position? (3 points)	
10. In triangle GHS, R is the midpoint of HS, point C is the midpoint of GR, M is the midpoint of HC, and L is the midpoint of MS. What is the ratio of the area of $\triangle GHS$ to $\triangle CML$? (3 points)	
11. Johnny and Kevin, best friends, are planning to meet in space. Johnny, returning from Zoho, is a high-ranking ambassador and has a luxurious spaceship that flies at 2750 miles per second. Kevin, who just got out of jail, has a rusted, old ship that flies at only 250 miles per second. On the way to meet each other, Kevin and Johnny reflect the same radio wave, which travels at 180,000 miles per second, back and forth to each other without stopping. If Kevin and Johnny begin reflecting the wave when they are 500,000 miles apart, the distance the radio wave travels (in miles) can be written as $A \times 10^B$ in scientific notation. What is the product of A and B? (3 points)	
12. Buzz has a rod 13 units long that he needs to cut to fix a part of the space shuttle. He makes two random cuts in the rod. What is the probability that the three pieces of the rod can form a non-degenerate triangle? (4 points)	

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