



# Rocket City Math League

## Mercury Solutions

**2006-2007**  
**Round 2**

<p>1. If group AX has 12 members, BX has 16 members, and together they share five people, you can subtract 5 from both groups to determine the number of members that are only in one group. So, <math>12-5=7</math> and <math>16-5=11</math>. Adding these two results together leaves you with 18. You can also draw a Venn Diagram to find your solution.</p>	18																																
<p>2. Since the radius of A ends at the center of B, and a radius of B ends at the center of A, we know that the two spheres have equal radii and are therefore congruent. If the radius of A is 360, then the radius of B is also 360 and the diameter is 720.</p>	720																																
<p>3. If Mono travels at 42mph and has no problems, then it travels 2268 miles in 54 hours (divide 2268 by 42). If Bio travels 36mph and has no problems, then it travels 2268 miles in 63 hours (divide 2268 by 36). However, Mono breaks down for 5 hours (where it breaks down is irrelevant), so it takes a total of 59 hours to travel 2268 miles. The difference between the time for Mono and Bio is therefore 4 hours.</p>	4																																
<p>4. If you cut the pizza into a total of seven pieces, you need a minimum of three cuts. See diagram.</p> <div style="text-align: center;"> </div>	3																																
<p>5. Let x represent the number of desks in a row. So, <math>15x</math> [the original number of rows]=<math>10(x+4)</math> [the new number of rows with the number of extra desks that the problem specifies.]</p> <p><math>15x=10(x+4)</math>  <math>15x=10x+40</math>  <math>5x=40</math>  <math>X=8</math> So there are 8 desks in the row. However, the problem asks for the total number of desks, so <math>10(8+4)=120</math>.</p>	120																																
<p>6. If Anna guesses on a true-false question, she has a 50-50 chance of getting it right, so the probability of answering one problem correctly is <math>\frac{1}{2}</math>. However, because Anna guesses on all ten problems and each guess is independent, she has a <math>\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}}</math> chance of getting a perfect score, which is equal to <math>\frac{1}{1024}</math>.</p>	1/1024																																
<p>7. The next term in the Fibonacci sequence is found by finding the sum of the two previous terms. Knowing this, you can discover that on page 10, the number written is 1. On page 20, the number is also 1. On 30, the number is <math>1 + 1 = 2</math>. For the rest, simply continue the pattern as follows:</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Page</th><th>#</th><th>Page</th><th>#</th><th>Page</th><th>#</th><th>Page</th><th>#</th></tr> </thead> <tbody> <tr> <td>30</td><td>2</td><td>60</td><td>8</td><td>90</td><td>34</td><td>120</td><td>144</td></tr> <tr> <td>40</td><td>3</td><td>70</td><td>13</td><td>100</td><td>55</td><td>130</td><td>233</td></tr> <tr> <td>50</td><td>5</td><td>80</td><td>21</td><td>110</td><td>89</td><td>140</td><td>377</td></tr> </tbody> </table> <p>The 14<sup>th</sup> term of the Fibonacci sequence, which will be shouted out on page 140 is 377</p>	Page	#	Page	#	Page	#	Page	#	30	2	60	8	90	34	120	144	40	3	70	13	100	55	130	233	50	5	80	21	110	89	140	377	377
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40	3	70	13	100	55	130	233																										
50	5	80	21	110	89	140	377																										
<p>8. Ackamatriana=10%      Bryzothmana=35      Chryzweethma=20%</p> <p>Using proportions, we realize that Bryzothmana scored <math>100\%-10\%-20\%=70\%</math> of the total points and we know that 70% is 35. Therefore, <math>\frac{70}{100} = \frac{35}{x}</math> where x is the total number of points scored. Using this, we discover that <math>70x=3500</math>, so x is 50. Knowing that Chryzweethma scored 20% of 50 points, we now know that Chryzweethma scored 10 points.</p>	10																																

9.	If the Criminal is	Then the statement of				Phantom																																																																												
		Zurg Is	Phantom is	Effervescence is	Evil Lord Quackers is																																																																													
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ELQ	False	True	True	False																																																																														
<p>The problem states that there is exactly one true statement out of the four given. If Zurg is the criminal, there are two true statements (the Phantom's and ELQ's). If Effervescence is the criminal, there are three true statements (all but her own). If ELQ is the criminal, there are two true statements (Phantom's and Effervescence's). However if the Phantom is the criminal, there is only one true statement (ELQ's) and therefore only the Phantom satisfies the conditions specified in the problem.</p>																																																																																		
<p>10. Kilogian 1 can do the job in 3 years, so he has a work rate of <math>1/3</math> of a ship per year. Kilogian 2 can build the ship in 5 years, so he has a work rate of <math>1/5</math> of a ship per year. Kilogian 3 can create the same ship in 7 years, so he has a work rate of <math>1/7</math> of a ship per year. To find the time needed for all three Kilogians to build the ship together, you add the individual work rates and take the reciprocal, because the inverse of the work rate equals the time it takes to build one ship. So, the combined work rate is <math>1/3 + 1/5 + 1/7 = 71/105</math>, and the time needed to build the ship is <math>105/71</math> years.</p>						105/71																																																																												
<p>11. <math display="block">\begin{array}{r} 112221 \\ 10021221_3 \\ 212011_3 \\ + 1101221_3 \\ \hline 12120000_3 \end{array}</math></p> <p>You have to realize that adding in base three is just like adding in base ten except that once you get three in a column, you put 1 in the next column. Also, a six in one column means that you put a 2 in the next column. The long way to do it is to convert the entire problem to base ten, add it together, and then convert it back to base three. (Since the question asked for the answer in base 3, leaving off the subscript 3 at the end is fine.)</p>						12120000 <sub>3</sub> or 12120000																																																																												
<p>12. Notice the pattern for the cost of the ticket. Using the table below, we can observe the pattern of the ticket cost and then find the total cost for the range of numbers.</p> <table border="1" style="display: inline-table; margin-right: 20px;"> <thead> <tr> <th>Ticket Number</th> <th>Cost (\$)</th> </tr> </thead> <tbody> <tr><td>162826</td><td>25</td></tr> <tr><td>7</td><td>26</td></tr> <tr><td>8</td><td>27</td></tr> <tr><td>9</td><td>28</td></tr> <tr><td>162830</td><td>20</td></tr> <tr><td>1</td><td>21</td></tr> <tr><td>2</td><td>22</td></tr> <tr><td>3</td><td>23</td></tr> <tr><td>4</td><td>24</td></tr> <tr><td>5</td><td>25</td></tr> <tr><td>6</td><td>26</td></tr> <tr><td>7</td><td>27</td></tr> <tr><td>8</td><td>28</td></tr> <tr><td>9</td><td>29</td></tr> </tbody> </table> <table border="1" style="display: inline-table; margin-right: 20px;"> <thead> <tr> <th>Ticket Range</th> <th>Cost (\$)</th> </tr> </thead> <tbody> <tr><td>162840</td><td>21</td></tr> <tr><td>to</td><td>to</td></tr> <tr><td>162849</td><td>30</td></tr> <tr><td> </td><td> </td></tr> <tr><td>162850</td><td>22</td></tr> <tr><td>to</td><td>to</td></tr> <tr><td>162859</td><td>31</td></tr> <tr><td> </td><td> </td></tr> <tr><td>162860</td><td>23</td></tr> <tr><td>to</td><td>to</td></tr> <tr><td>162869</td><td>32</td></tr> <tr><td> </td><td> </td></tr> <tr><td>162870</td><td>24</td></tr> <tr><td>to</td><td>to</td></tr> <tr><td>162878</td><td>32</td></tr> </tbody> </table> <table border="1" style="display: inline-table;"> <thead> <tr> <th>Range of Numbers</th> <th>Cost of Range (\$)</th> </tr> </thead> <tbody> <tr><td>162826-162829</td><td>106</td></tr> <tr><td>162830s</td><td>245</td></tr> <tr><td>162840s</td><td>255</td></tr> <tr><td>162850s</td><td>265</td></tr> <tr><td>162860s</td><td>275</td></tr> <tr><td>162870-162878</td><td>252</td></tr> </tbody> </table>						Ticket Number	Cost (\$)	162826	25	7	26	8	27	9	28	162830	20	1	21	2	22	3	23	4	24	5	25	6	26	7	27	8	28	9	29	Ticket Range	Cost (\$)	162840	21	to	to	162849	30			162850	22	to	to	162859	31			162860	23	to	to	162869	32			162870	24	to	to	162878	32	Range of Numbers	Cost of Range (\$)	162826-162829	106	162830s	245	162840s	255	162850s	265	162860s	275	162870-162878	252	\$284
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<p>Okay, so now we have figured out how much each range of numbers costs. Applying what we know, the amount sold was \$1114.00, and the sum of the costs of each range of ticket numbers is \$1398. To find the value of the unsold tickets, simply subtract the amount sold from the total value, resulting in <math>1398 - 1114 = \\$284</math>.</p>																																																																																		

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