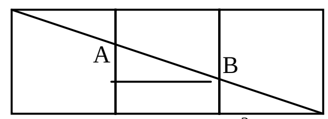
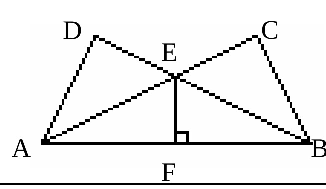
Rocket City Math League

Gemini Solutions

2006-2007
Round 2

1. The area of the circle is $r^2\pi = 3^2\pi = 9\pi$	**9π**
2. Drawing a regular tetrahedron, we can see that the solid has 6 edges.	**6**
3. Finding the area of the equilateral triangle, we get $\frac{(8)^2\sqrt{3}}{4}$, or $16\sqrt{3}$. Since the isosceles right triangle has a hypotenuse of length 8, its legs have length $\frac{8}{\sqrt{2}}$, or $4\sqrt{2}$. The isosceles triangle then has area of $\left(\frac{1}{2}\right)(4\sqrt{2})(4\sqrt{2})$, or 16. The area of the total figure is $16+16\sqrt{3}$.	$16+16\sqrt{3}$
4. The original cone has a volume of $\left(\frac{1}{3}\right)(24)(16\pi)$, or 128π. So $128\pi = (12)\left(\frac{4}{3}\pi r^2\right)$, and $r = 2$.	**2**
5. Using similar triangles, we see that the distance from A to the bottom side of the rectangle is $\frac{2}{3}$, and the distance from B to the same side is $\frac{1}{3}$. So the legs of the new right triangle shown in the figure to the right are $\frac{1}{3}$ and 1. Using the Pythagorean Theorem, we get $\left(\frac{1}{3}\right)^2 + 1^2 = x^2$, so $x = \frac{\sqrt{10}}{3}$. 	$\frac{\sqrt{10}}{3}$
6. There are 16 squares of side length 1, 9 squares of side length 2, 4 squares of side length 3, and 1 square with side length 4. Now looking at squares formed by non adjacent dots, there are 9 squares with side length $\sqrt{2}$, 8 squares with side length $\sqrt{5}$, 2 squares with side length $\sqrt{10}$, and 1 square with side length $2\sqrt{2}$. So there are $16+9+4+1+9+8+2+1=50$ squares.	**50**
7. Since there are 360 degrees in a circle, we get $3x+4x+5x+8x=360$, so $x=18$. The smallest angle of the quadrilateral is equal to ½ of the sum of the two smallest consecutive acrs in the circle, so the measure of the angle is $\left(\frac{1}{2}\right)(7\cdot 18)$, or 63 degrees.	**63**
8. The folded piece of paper is shown below. Using the Pythagorean Theorem to solve for the diagonal, we obtain $AB = 4\sqrt{5}$. From the given information, $DE + EB = 8$. Dropping perpendicular bisector $\overline{EF}$ from the intersection point of the two sides with length 8, we see that $\frac{FB}{EB} = \frac{DB}{AB}$. Solving for EB, $\frac{2\sqrt{5}}{EB} = \frac{8}{4\sqrt{5}}$, $EB = 5$, and $DE = 3$. The total area is the area of $\Delta ADE + \Delta ACB$, so $\left(\frac{1}{2}\right)(3)(4) + \left(\frac{1}{2}\right)(4)(8) =$ 22. 	**22**
9. $x(x-8) = -9 - y(y-6) \rightarrow x^2 - 8x + y^2 - 6y = -9 \rightarrow x^2 - 8x + 16 + y^2 - 6y + 9 = 16 \rightarrow (x-4)^2 + (y-3)^2 = 4^2$ The graph of this equation is a circle with center (4,3) with a radius of 4. One can find that the positive integers in the domain are: 1, 2, 3, 4, 5, 6, 7, and 8, and the positive integers in the range are 1, 2, 3, 4, 5, 6, and 7. Adding these, we get 64.	**64**

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10. The centroid for this triangle is $\left(\frac{-2+-5+-1}{3},\frac{4+-2+-4}{3}\right)$, or $(-\frac{8}{3},-\frac{2}{3})$. By graphing, we find that the hypotenuse is the line between the points $(-2,4)$ and $(-1,-4)$, and the equation of this line is $8x+y+12=0$. The distance from a point (x_0, y_0) to a line $Ax+By+C=0$ is defined as: $\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$. Plugging in the variables and rationalizing the denominator, the distance is $\frac{2\sqrt{65}}{13}$.

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$\frac{2\sqrt{65}}{13}$

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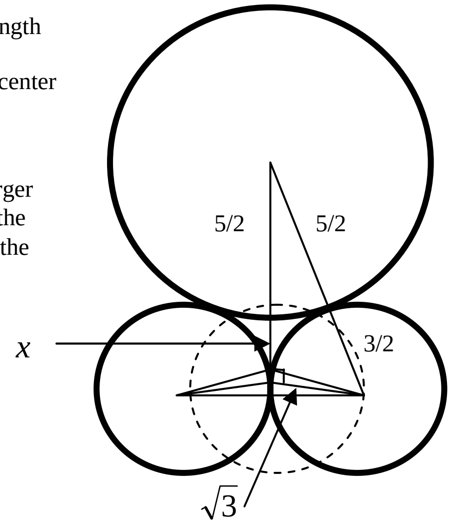
11. To begin, we connect the centers of the three smaller spheres with radii of length 3/2.
This forms an equilateral triangle with side length 3. The distance from the center of the triangle to a vertex is then $\frac{3}{\sqrt{3}}$, or $\sqrt{3}$.
Now connecting any vertex of the equilateral triangle to the center of the larger sphere, a right triangle forms, as shown in the figure to the right. Let x be the shortest distance from the center of the equilateral triangle to the surface of the larger sphere.
Using the Pythagorean Theorem:

$$\left(\sqrt{3}\right)^2+\left(x+5/2\right)^2=\left(5/2+3/2\right)^2$$
$$3+\left(x+5/2\right)^2=16$$
$$\left(x+5/2\right)^2=13$$
$$x=-\frac{5}{2}\pm\sqrt{13}$$



Since the distance must be positive, $x=-\frac{5}{2}+\sqrt{13}$. The distance from the top of the sphere to the plane is:

$$\frac{5}{2}+\frac{5}{2}+\left(-\frac{5}{2}+\sqrt{13}\right)+\frac{3}{2}, \text{ or } 4+\sqrt{13}.$$

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$4+\sqrt{13}$

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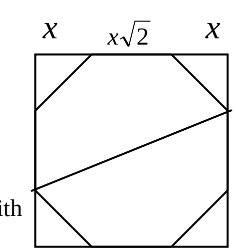
12. Shown to the right is regular octagon SUPERMAN sharing sides with a square. Let the four isosceles at the corner of the square have leg length x. The hypotenuse then has length $x\sqrt{2}$. Since the octagon is regular, the side of octagon on the square also has length $x\sqrt{2}$.



We are given $UM=8\sqrt{2}$. Drawing square UNME, we see that UM is a diagonal, so $UN=NM=ME=EU=8$. Focusing on side UN, UN is the hypotenuse of a right triangle with legs x and $x+x\sqrt{2}$. Using the Pythagorean Theorem:

$$x^2+\left(x+x\sqrt{2}\right)^2=64$$
$$=x^2+x^2+2x^2\sqrt{2}+2x^2$$
$$=4x^2+2x^2\sqrt{2}=8^2$$
$$=2x^2\left(2+\sqrt{2}\right)=64$$

= Solving for x^2, we get $32-16\sqrt{2}$.

Area of the square – 4(Area of isosceles triangle):

$$\left(2x+x\sqrt{2}\right)^2-4\left(\frac{1}{2}x^2\right)=4x^2+4x^2\sqrt{2}+2x^2-2x^2=4x^2+4x^2\sqrt{2}=4\left(x^2+x^2\sqrt{2}\right)$$

Substituting our obtained value for x^2, we get $4\left\{32-16\sqrt{2}+\sqrt{2}\left(32-16\sqrt{2}\right)\right\}=64\sqrt{2}$.

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$64\sqrt{2}$

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