



Rocket City Math League Discovery Test

**2005-2006
Round 3**

Answers must be written inside the adjacent answer boxes. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as improper fractions (unless requested otherwise in the problem).

No calculators, books, or other aids may be used.

1. At a particular high school there are 1500 students of which 60% are boys. 45% of the boys and 40% of the girls are in Algebra-II or higher. How many students are in math classes Algebra-II or higher? (1 point)	
2. On the newly-discovered planet Hercules, a dangerous bacteria multiplies rapidly. Its population doubles every 4 days. If on August 1 there were 15 bacteria, how many bacteria are there at the same time of day on September 2 of the same year? (1 point)	
3. Alex has discovered a chemical solution, composed of Zyron and Grutak. Alex has a 12 L solution composed of 20% Zyron and 80% Grutak. How many liters of this solution must be removed and replaced with pure Zyron, to make a solution which is 60% Zyron? (1 point)	
4. In the following sequence of positive integers 1,2,2,3,3,3,4,4,4,4,5,5,5,5,... the n^{th} positive integer appears n times. What is the remainder when the 95 th term is divided by 10? (1 point)	
5. An equilateral triangle is inscribed in a circle with circumference 6π . If the area of the triangle is in the form $a\sqrt{b}$ in simplified form, find $[a + b]$, where $[n]$ is the greatest integer less than or equal to n . (2 points)	
6. Suppose that 1 and 3 are two roots of the equation $f(x) = x^3 + ax^2 + bx + c$ and that $a + b = -13$. What is the third root? (2 points)	
7. For which <u>positive</u> integers, a , will $(48 + a^2)$ be a perfect square? (2 points)	
8. In how many distinct ways can 8 different keys be arranged on a keychain? (Two arrangements of keys are not considered different if the keys are in the same order or can be made to be in the same order without removing keys from the keychain.) (2 points)	
9. The flight of a rocket is mapped by the parametric equations below, where $x(t)$ is the horizontal position and $y(t)$ is the vertical position. Find the horizontal position at which the rocket reaches its maximum height. (3 points) $x(t) = 20(t-1) \quad y(t) = -t^2 + 22t + 23$	
10. A is the set of foci of the hyperbola of the equation $16x^2 + 128x - 9y^2 + 180y = 788$, and B is the set of foci of the ellipse of the equation $3y^2 + 78y + 4x^2 + 48x = -639$. If an element from A and an element from B are selected at random, what is the maximum possible distance between the two points? (3 points)	
11. Find the volume when the region enclosed by the graphs of $y \leq - x + 2 - x - 3 + 8$ and $y > 0$ is rotated about the x-axis. (3 points)	
12. Evaluate: $\sum_{i=2}^{2006} \frac{i^2 + 1}{(i-1)(i)(i+1)}$. (4 points)	

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