Rocket City Math League 2005-2006
Apollo Solutions Round 2

1. Answer: **<u>41/3</u>**

$$2 \cdot 3 + 4(2 \div 3) + 5$$
$$= 6 + 8/3 + 5 = 41/3$$

2. Answer: **<u>48</u>**

Let x be the number of students in both classes. So,

$$x = \frac{1}{4}\left((30-x)+(30-x)+x\right)$$
$$4x = 60 - x$$
$$5x = 60$$
$x = 12$: the number of students in both classes

Thus the total number of students is 2(30-12)+12=48

3. Answer: **-8/11**

To solve this problem, let a=1/x and b=1/y. We now have the following equations:
a + b = 6
2a + 3b = -4
These equations can then be solved using elimination and substitution, and we find that a=22 and b=-16. Therefore, x=1/22 and y=-1/16.
Thus x/y = $-\frac{8}{11}$

4. Answer: **<u>200</u>**

There are ${}_5C_2$ = 10 ways to choose 2 of the 5 CEOs. There are ${}_6C_3$ = 20 ways to choose 3 of the 6 Chief Engineers. Therefore, there are 10 x 20 = 200 ways to choose 2 CEOs and 3 Chief Engineers.

5. Answer: **<u>0</u>**

Let $x = 2^y$
$$3(2x^2) - 5x - 1 = 0$$
$$6x^2 - 5x - 1 = 0$$
$$(6x+1)(x-1) = 0$$
$$x = -\frac{1}{6}, 1$$
Solve for y, only the positive x-value has a y-value
$$1 = 2^y$$
$$y = 0$$

6. Answer: **<u>9</u>**

By using the change of base rule, we find that $\log_9 x = \frac{1}{2}\log_3 x$. Also,

$1 = \log_3 3$, so the equation can be written as $\log_3 x = \frac{3}{2}\log_3 x - \log_3 3$.

Combining like terms: $\frac{1}{2}\log_3 x = \log_3 3$, or $\log_3 \sqrt{x} = \log_3 3$. It follows that

$\sqrt{x} = 3$, so x=9

7. Answer: **<u>10</u>**

Basically, this problem is asking for the number of lattice points in the region bound by the equations x = 0, y = 0, and 6x + 5y = 30. The lattice points that are in this region will be in the first quadrant, and will lie below the line 6x + 5y = 30. So, the coordinates of the "meteorites" must be positive integers that satisfy the inequality 6x + 5y < 30.
When x = 1, y < 4.8; the lattice points at x = 1 are (1, 1), (1, 2), (1, 3), and (1, 4).
x = 2, y < 3.6; (2, 1), (2, 2), (2, 3)
x = 3, y < 2.4; (3, 1), (3, 2)
x = 4, y < 1.1; (4, 1)
There are a total of 10 lattice points that lie in this region.

8. Answer: **<u>7</u>**

We can calculate the number of 0's at the end of "10!9!8!7!6!5!4!3!2!1!" by counting the number of 5's. There is one five in 5!, 6!, 7!, 8!, and 9!, and there are two 5's in 10! (10 = 5 x 2). This gives us a total of 7 fives, and therefore there are 7 zeroes at the end of "10!9!8!7!6!5!4!3!2!1!".

9. Answer: **9/128**

The probability that there will be a quiz is 1/8. The probability that Nitsuj will stay awake when there is a quiz is 1- 7/16 = 9/16. Therefore, the probability that there will be a quiz and Nitsuj will stay awake is 1/8 x 9/16 = 9/128.

10. Answer: **104**

When we look at the graph of $12|x|+5|y|=60$, we come to the conclusion that it doesn't matter where we start; we can simply choose any point and pass through that point twice. If we choose a point and then travel around the graph so that we have passed the beginning point twice, we will have gone around the graph two times, so the distance we will have traveled will be twice the perimeter of the figure. The perimeter is 13 x 4 = 52, so twice the perimeter is 104.

11. Answer: **1250**

Without a loss of generality, the first train is not moving and the second train with length l ft is moving at 350 fpm toward train 1.

$distance = rate \cdot time$

$d = 350 \cdot 5 = 1750 = l + 500$

So $l = 1250$

12. Answer: **9**

The prime factorization of 153 is $9 \cdot 17$, so we can take the (mod 9) of the diophantine equation. Substitute $a = x - 1$ and $b = y - 3$ to simplify the equation. The new equation will be $(a^3 + 153b^3 = 63)(\text{mod} 9)$. Consider the (mod 9) of the cubes: $1^3 \equiv 1$, $2^3 = 8 \equiv -1$, $3^3 \equiv 0$, $4^3 = 64 \equiv 1$, $5^3 = 125 \equiv -1$, $6^3 \equiv 0$, $7^3 = (-2)^3 \equiv 1$, $8^3 = (-1)^3 = -1$, $9^3 \equiv 0$. We can see that the (mod 9) of any cube is 1, -1, or 0. Because $153 \equiv 0 (\text{mod } 9)$, the equation is $a^3 \equiv 0 (\text{mod} 9)$. Thus, for integer a, $a^3 = 0, 27, 216, \ldots$, therefore, $a = 0, 3, 6, \ldots$ or $x = 1, 4, 7, \ldots$ The first positive integral (x, y) is $(7, 2)$, so x+y=9