Rocket City Math League

2008-2009

Senior Division

Inter-School Test

Answers must be written on the separate official answer sheet. All answers must be written in exact, reduced, simplified, and rationalized form. All decimals and mixed numbers must be written as improper fractions unless otherwise specified. **No calculators, books, or other aides may be used. You will be allowed 30 minutes to complete the test. Each team may only submit one official answer sheet.**

1. Find the sum of the digits in the product: $12345 \cdot 54321$. *(1 point)*

2. In how many distinct ways can the letters in the word "EINSTEIN" be arranged? *(1 point)*

3. Jim and Casey each roll a fair six-sided die. If A is the probability that they both roll prime numbers, and B is the probability that they both roll composite numbers, find A+B. *(1 point)*

4. If f(x)= $x \cdot$(the sum of the digits in the base ten representation of x), find the sum of the digits in $f(f(f(f(f(f(2))))))$. *(1 point)*

5. Find the product of the magnitudes of the imaginary solutions to the equation: $x^3 - x^2 + 2 = 0$. *(1 point)*

6. Patrick loves equilateral triangles. He starts with a single equilateral triangle with side length 1 that was drawn by his brother and connects the midpoints of this triangle to form a smaller triangle. He then connects the midpoints of this smaller triangle to form an even smaller triangle. He repeats this same process of connecting the midpoints of the last triangle he created until finally he gets bored and leaves to go watch television. If, in total, he drew 12 line segments, what is the area enclosed by the smallest triangle that he formed? *(2 points)*

7. Sarah, John, Barack, and Joe love to eat scones. Being fair and just people, they decide to take the 16 identical scones that were donated to them and divide them among themselves so that each person has at least two scones. In how many different ways can they divide the scones? *(2 points)*

8. What is the units digit of the sum: $2007^{2008} + 2008^{2009}$? *(2 points)*

9. What is the 7th prime number in the Fibonacci sequence? *(2 points)*

10. Find the volume of the largest cone that can fit completely inside a square pyramid with a volume of 196. *(3 points)*

11. Zippy the Zebra is tied with a rope of length 10 to the outside of a corner of a barn. If the barn is in the shape of a regular hexagon with side length 5, then what is the area of the region outside the barn within which Zippy can walk? (Note: A corner of the barn is a vertex of the hexagon.) *(3 points)*

12. If a and b are the two real solutions to the equation: $x^2 - 132x + 36 = 0$, and $n = a\sqrt{b} + b\sqrt{a}$, then find n, given that n must be an integer. *(3 points)*

13. What is the sum of the infinite series: $\frac{1}{2}+\frac{1}{2}+\frac{3}{8}+\frac{1}{4}+\frac{5}{32}+\frac{3}{32}+\ldots$? *(4 points)*

14. Find the product of the 29th digit past the decimal point in the decimal expansion of $\frac{5}{98}$ and the 49th digit past the decimal point in the decimal expansion of $\frac{1}{198}$. *(4 points)*

15. A triangle, a rectangle, a pentagon, and a hexagon, all of which are convex polygons, intersect in a finite number of points in a single plane. What is the maximum possible number of points of intersection among them? *(5 points)*