Potpourri **2008 Northwest Mathematics Championship**

Answers must be exact, or have four or more significant digits, correctly rounded, unless specified otherwise in a particular problem.

1. Find the sum of all the positive integers whose squares are less than 2008.

2. N is a four digit number which is a multiple of 9, and its digits, taken in order from left to right are in a descending arithmetic sequence. What are the possible values of N?

3. The following numbers (written in base n, $n + 1$, and $n + 2$) are in arithmetic progression: 23_n, 29_{n+1}, 32_{n+2}. What is the value of n?

4. A circle is inscribed in a square, and a square is then inscribed in that circle. If a point is randomly selected within the original square, what is the probability that it also lies on the interior of the circle, but not the interior of the second square? Give your answer to the nearest thousandth.

5. The product of two positive numbers is 192. The sum of their squares is 400. What is the sum of the numbers?

6. John started this morning writing down positive integers in order, starting with the number 1. If John has written 2008 digits so far, what are the next two digits John will write?

7. A digital watch reads in 12 hour time, and its current time display is J:KL, where J, K and L are distinct one digit integers. In 50 minutes it will read K:JL, and 236 minutes from then it will read L:JK. What is the value of the following expression: $L + K - 2J$?

8. A box contains 64 crayons, which are divided evenly into shades of green, shades of red, shades of yellow, and shades of gray. If half the green crayons are removed, along with a quarter of the red crayons, and 3 yellow crayons, what is the probability that the next two crayons drawn from the box will be the same color? Give your answer as a fraction in lowest terms.

9. Nine dots are drawn on a page. These nine dots are: the corners of a square, the midpoints of the sides of the square, and the intersection of the diagonals of the square. Using some of these points as endpoints, two distinct line segments are randomly drawn. What is the probability that the two segments are collinear?

10. An airplane takes two minutes longer to complete a journey when it is traveling against the wind than when it is traveling with the wind. If the wind speed was cut in half, it would take 0.99 minutes longer to complete the journey when it is traveling against the wind than when it is traveling with the wind. What is the ratio of the airplane speed (in still air) to the original wind speed, to the nearest thousandth?