

# I2MC Team Round

March 26, 2011

Write all answers on the answer sheet, in terms of common fractions, radicals, and/or pi when necessary.

1. The product of three prime numbers is equal to their sum multiplied by 5. Find these three prime numbers.
2. A right triangle has legs with lengths of 3 and 4. What is the ratio of the areas of the circumcircle to the incircle?
3. What is the 37th item in an alphabetized list of all the permutations of the word PUNEET?
4. Helen and Matthew both want coffee between 3pm and 4pm. If they each visit Starbucks for 10 minutes, what is the probability that they will see each other there?
5. How many 3 digit numbers exist where the difference between the largest digit and the smallest digit is no larger than 1?
6. If  $a^x = c^q$  and  $c^y = a^z$ , then express  $x$  in terms of  $q$ ,  $z$ , and  $y$ .
7. Real numbers  $x$  and  $y$  satisfy  $x^2 + y^2 = 1$ . Find the maximum value for  $4x + 3y$ .
8. What is the smallest positive integer that has exactly 12 positive integer factors and leaves a remainder of 3 when divided by 5?
9. Find the number of integer points  $(x, y)$  in the region between  $y = x^2$  and  $y = 100$ , including those lying on  $y = x^2$  and  $y = 100$ .
10. A train, an hour after starting, meets with an accident which detains it a half hour, after which it proceeds at  $\frac{3}{4}$  of its former rate and arrives 3.5 hours late. Had the accident happened 90 miles farther along the line, it would have arrived only 3 hours late. What was the length of the trip in miles?
11. If  $a$ ,  $b$  and  $c$  are positive integers where  $c = (a + bi)^3 - 107i$  and  $i = \sqrt{-1}$ , find  $c$ .
12. I answer false to a truth/false question. Every minute, I have a  $\frac{3}{4}$  probability of changing my answer. If there are 6 minutes left on the test, what is the probability that I will end up answering false when time is up?
13. A pyramid has a square base with sides of length  $\sqrt{2}$  and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?
14. How many ways can 4 members of a math team sit in a row of 12 chairs in a row such that no two mathletes are sitting next to each other?
15. Let ABCD be a square, and let M and N be the midpoints of BC and CD, respectively. Find  $\sin(\angle MAN)$ .
16. What is the probability that 3 real numbers (including non-integers) between 0-10 add up to 8 or less?