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American Mathematics Competitions



19th Annual

AMC 8

(American Mathematics Contest 8)

Solutions Pamphlet

Tuesday, NOVEMBER 18, 2003

This Solutions Pamphlet gives at least one solution for each problem on this year's exam and shows that all the problems can be solved using material normally associated with the mathematics curriculum for students in eighth grade or below. These solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

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1. **(E)** A cube has 12 edges, 8 corners and 6 faces. The sum is 26.
2. **(C)** The smallest prime is 2, which is a factor of every even number. Because 58 is the only even number, it has the smallest prime factor.
3. **(D)** Since 30 of the 120 grams are filler, $\frac{30}{120} = 25\%$ of the burger is filler. So $100\% - 25\% = 75\%$ of the burger is not filler.

OR

There are $120 - 30 = 90$ grams that are not filler. So $\frac{90}{120} = 75\%$ is not filler.

4. **(C)** The following chart shows that the answer must be 5 tricycles.

Bicycles	Tricycles	Wheels
0	7	21
1	6	20
2	5	19
3	4	18

OR

Let b equal the number of bicycles and t equal the number of tricycles. Then the number of vehicles is $b + t = 7$, and the number of wheels is $2b + 3t = 19$. Because $b = 7 - t$, it follows that

$$2(7 - t) + 3t = 19$$

$$14 - 2t + 3t = 19$$

$$14 + t = 19$$

$$t = 5.$$

OR

If each child had a bicycle, there would be 14 wheels. Since there are 19 wheels, 5 of the vehicles must be tricycles.

5. **(B)** If 20% of the number is 12, the number must be 60. Then 30% of 60 is $0.30 \times 60 = 18$.

OR

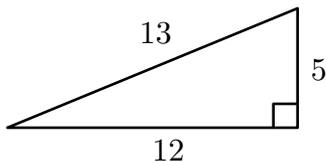
Since 20% of the number is 12, it follows that 10% of the number is 6. So 30% of the number is 18.

6. (B)

$$A = \frac{1}{2}(\sqrt{144})(\sqrt{25})$$

$$A = \frac{1}{2} \cdot 12 \cdot 5$$

$$A = 30 \text{ square units}$$



7. (A) Blake scored a total of $4 \times 78 = 312$ points on the four tests. Jenny scored $10 - 10 + 20 + 20 = 40$ more points than Blake, so her average was $\frac{352}{4} = 88$, or 10 points higher than Blake's.

OR

The total point difference between Jenny's and Blake's tests was $10 - 10 + 20 + 20 = 40$ points. The average difference is $\frac{40}{4} = 10$ points.

8. (A) Because all of the cookies have the same thickness, only the surface area of their shapes needs to be considered. The surface area of each of Art's trapezoid cookies is $\frac{1}{2} \cdot 3 \cdot 8 = 12 \text{ in}^2$. Since he makes 12 cookies, the surface area of the dough is $12 \times 12 = 144 \text{ in}^2$.

Roger's rectangle cookies each have surface area $2 \cdot 4 = 8 \text{ in}^2$; therefore, he makes $144 \div 8 = 18$ cookies.

Paul's parallelogram cookies each have surface area $2 \cdot 3 = 6 \text{ in}^2$. He makes $144 \div 6 = 24$ cookies.

Trisha's triangle cookies each have surface area $\frac{1}{2} \cdot 4 \cdot 3 = 6 \text{ in}^2$. She makes $144 \div 6 = 24$ cookies.

So Art makes the fewest cookies.

9. (C) Art's 12 cookies sell for $12 \times \$0.60 = \7.20 . Roger's 18 cookies should cost $\$7.20 \div 18 = \0.40 each.

OR

The trapezoid's area is 12 in^2 and the rectangle's area is 8 in^2 . So the cost of a rectangle cookie should be $\left(\frac{8}{12}\right) 60\text{¢} = 40\text{¢}$.

10. (E) The triangle's area is 6 in^2 , or half that of the trapezoid. So Trisha will make twice as many cookies as Art, or 24.

11. **(B)** Thursday's price of \$40 is increased 10% or \$4, so on Friday the shoes are marked \$44. Then 10% of \$44 or \$4.40 is taken off, so the price on Monday is $\$44 - \$4.40 = \$39.60$.

OR

Marking a price 10% higher multiplies the original price by 1.1, and reducing a price by 10% multiplies the price by 0.9. So the price of a pair of shoes that was originally \$40 will be $\$40 \times 1.1 \times 0.9 = \39.60 .

12. **(E)** If 6 is one of the visible faces, the product will be divisible by 6. If 6 is not visible, the product of the visible faces will be $1 \times 2 \times 3 \times 4 \times 5 = 120$, which is also divisible by 6. Because the product is always divisible by 6, the probability is 1.
13. **(B)** A cube has four red faces if it is attached to exactly two other cubes. The four top cubes are each attached to only one other cube, so they have five red faces. The four bottom corner cubes are each attached to three others, so they have three red faces. The remaining six each have four red faces.
14. **(D)** As given, $T = 7$. This implies that $F = 1$ and that O equals either 4 or 5. Since O is even, $O = 4$. Therefore, $R = 8$. Replacing letters with numerals gives

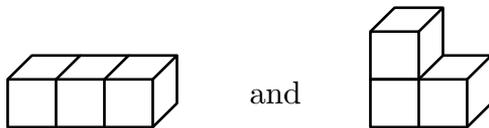
$$\begin{array}{r} 7 \quad W \quad 4 \\ + \quad 7 \quad W \quad 4 \\ \hline 1 \quad 4 \quad U \quad 8 \end{array}$$

$W + W$ must be less than 10; otherwise, a 1 would be carried to the next column, and O would be 5. So $W < 5$. $W \neq 0$ because $W \neq U$, $W \neq 1$ because $F = 1$, $W \neq 2$ because if $W = 2$ then $U = 4 = O$, and $W \neq 4$ because $O = 4$. So $W = 3$.

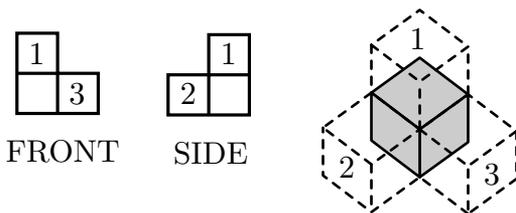
The addition problem is

$$\begin{array}{r} 7 \quad 3 \quad 4 \\ + \quad 7 \quad 3 \quad 4 \\ \hline 1 \quad 4 \quad 6 \quad 8 \end{array}$$

15. **(B)** There are only two ways to construct a solid from three cubes so that each cube shares a face with at least one other:



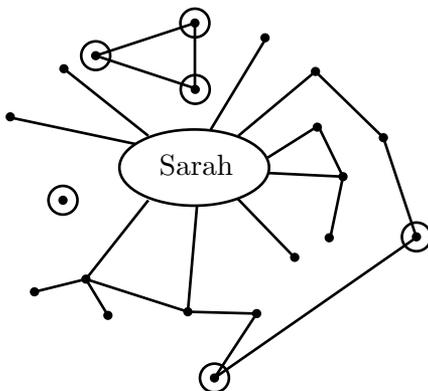
Neither of these configurations has both the front and side views shown. The four-cube configuration has the required front and side views. Thus at least four cubes are necessary.



Question: Is it possible to construct a five-cube configuration with these front and side views?

16. **(D)** There are 2 choices for the driver. The other three can seat themselves in $3 \times 2 \times 1 = 6$ different ways. So the number of seating arrangements is $2 \times 6 = 12$.
17. **(E)** Because Jim has brown eyes and blond hair, none of his siblings can have both blue eyes and black hair. Therefore, neither Benjamin nor Tevyn can be Jim's sibling. Consequently, there are only three possible pairs for Jim's siblings – Nadeen and Austin, Nadeen and Sue, or Austin and Sue. Since Nadeen has different hair color and eye color from both Austin and Sue, neither can be Nadeen's sibling. So Austin and Sue are Jim's siblings. Benjamin, Nadeen and Tevyn are siblings in the other family.

18. (D) In the graph below, the six classmates who are not friends with Sarah or with one of Sarah's friends are circled. Consequently, six classmates will not be invited to the party.

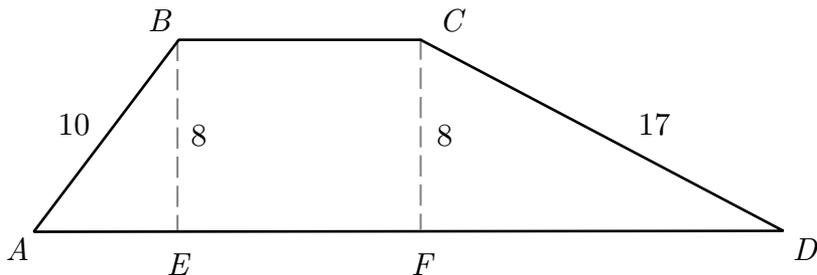


19. (C) A number with 15, 20 and 25 as factors must be divisible by their least common multiple (LCM). Because $15 = 3 \times 5$, $20 = 2^2 \times 5$, and $25 = 5^2$, the LCM of 15, 20 and 25 is $2^2 \times 3 \times 5^2 = 300$. There are three multiples of 300 between 1000 and 2000: 1200, 1500 and 1800.
20. (D) Note that in the same period of time, the hour hand moves $\frac{1}{12}$ as far as the minute hand. At 4:00 a.m., the minute hand is at 12 and the hour hand is at 4. By 4:20 a.m., the minute hand has moved $\frac{1}{3}$ of way around the clock to 4, and the hour hand has moved $\frac{1}{12} \times \frac{1}{3} = \frac{1}{36}$ of the way around the clock from 4. Therefore, the angle formed by the hands at 4:20 a.m. is $\frac{1}{36} \cdot 360^\circ = 10^\circ$.

OR

As the minute hand moves $\frac{1}{3}$ of the way around the clock face from 12 to 4, the hour hand will move $\frac{1}{3}$ of the way from 4 to 5. So the hour hand will move $\frac{1}{3}$ of $\frac{1}{12}$ of 360° , or 10° .

21. **(B)** Label the feet of the altitudes from B and C as E and F respectively. Considering right triangles AEB and DFC , $AE = \sqrt{10^2 - 8^2} = \sqrt{36} = 6$ cm, and $FD = \sqrt{17^2 - 8^2} = \sqrt{225} = 15$ cm. So the area of $\triangle AEB$ is $\frac{1}{2}(6)(8) = 24$ cm², and the area of $\triangle DFC$ is $(\frac{1}{2})(15)(8) = 60$ cm². Rectangle $BCFE$ has area $164 - (24 + 60) = 80$ cm². Because $BE = CF = 8$ cm, it follows that $BC = 10$ cm.



OR

Let $BC = EF = x$. From the first solution we know that $AE = 6$ and $FD = 15$. Therefore, $AD = x + 21$, and the area of the trapezoid $ABCD$ is $(8) \left\{ \frac{1}{2} [x + (x + 21)] \right\} = 164$. So

$$4(2x + 21) = 164,$$

$$2x + 21 = 41,$$

$$2x = 20,$$

and $x = 10$.

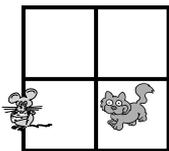
22. **(C)** For Figure A, the area of the square is $2^2 = 4$ cm². The diameter of the circle is 2 cm, so the radius is 1 cm and the area of the circle is π cm². So the area of the shaded region is $4 - \pi$ cm².

For Figure B, the area of the square is also 4 cm². The radius of each of the four circles is $\frac{1}{2}$ cm, and the area of each circle is $(\frac{1}{2})^2 \pi = \frac{1}{4}\pi$ cm². The combined area of all four circles is π cm². So the shaded regions in A and B have the same area.

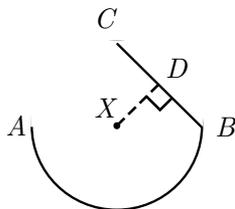
For Figure C, the radius of the circle is 1 cm, so the area of the circle is π cm². Because the diagonal of the inscribed square is the hypotenuse of a right triangle with legs of equal lengths, use the Pythagorean Theorem to determine the length s of one side of the inscribed square. That is, $s^2 + s^2 = 2^2 = 4$. So $s^2 = 2$ cm², the area of the square. Therefore, the area of the shaded region is $\pi - 2$ cm². Because $\pi - 2 > 1$ and $4 - \pi < 1$, the shaded region in Figure C has the largest area.

Note that the second figure consists of four small copies of the first figure. Because each of the four small squares has sides half the length of the sides of the big square, the area of each of the four small figures is $\frac{1}{4}$ the area of Figure A. Because there are four such small figures in Figure B, the shaded regions in A and B have the same area.

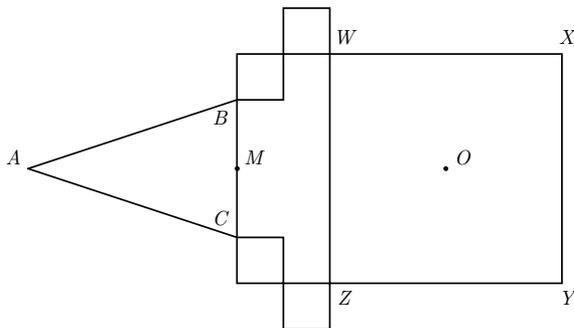
23. (A) There are four different positions for the cat in the 2×2 array, so after every fourth move, the cat will be in the same location. Because $247 = 4 \times 61 + 3$, the cat will be in the 3rd position clockwise from the first, or the lower right quadrant. There are eight possible positions for the mouse. Because $247 = 8 \times 30 + 7$, the mouse will be in the 7th position counterclockwise from the first, or the left-hand side of the lower left quadrant.



24. (B) All points along the semicircular part of the course are the same distance from X , so the first part of the graph is a horizontal line. As the ship moves from B to D , its distance from X decreases, then it increases as the ship moves from D to C . Only graph B has these features.



25. (C) Let M be the midpoint of \overline{BC} . Since $\triangle ABC$ is isosceles, \overline{AM} is an altitude to base \overline{BC} . Because A coincides with O when $\triangle ABC$ is folded along \overline{BC} , it follows that $AM = MO = \frac{5}{2} + 1 + 1 = \frac{9}{2}$ cm. Also, $BC = 5 - 1 - 1 = 3$ cm, so the area of $\triangle ABC$ is $\frac{1}{2} \cdot BC \cdot AM = \frac{1}{2} \cdot 3 \cdot \frac{9}{2} = \frac{27}{4}$ cm².



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