

Wednesday, FEBRUARY 25, 2004

55th Annual American Mathematics Contest 12

AMC 12



Contest B

**The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions**

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have 75 MINUTES working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 22nd annual American Invitational Mathematics Examination (AIME) on Tuesday, March 23, 2004 or Tuesday, April 6, 2004. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

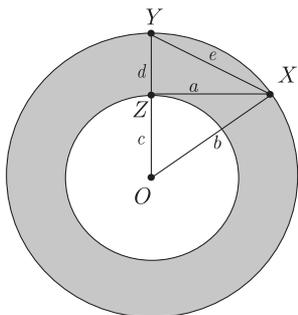
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- At each basketball practice last week, Jenny made twice as many free throws as she made at the previous practice. At her fifth practice she made 48 free throws. How many free throws did she make at the first practice?
(A) 3 (B) 6 (C) 9 (D) 12 (E) 15
- In the expression $c \cdot a^b - d$, the values of a , b , c , and d are 0, 1, 2, and 3, although not necessarily in that order. What is the maximum possible value of the result?
(A) 5 (B) 6 (C) 8 (D) 9 (E) 10
- If x and y are positive integers for which $2^x 3^y = 1296$, what is the value of $x + y$?
(A) 8 (B) 9 (C) 10 (D) 11 (E) 12
- An integer x , with $10 \leq x \leq 99$, is to be chosen. If all choices are equally likely, what is the probability that at least one digit of x is a 7?
(A) $\frac{1}{9}$ (B) $\frac{1}{5}$ (C) $\frac{19}{90}$ (D) $\frac{2}{9}$ (E) $\frac{1}{3}$
- On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d ?
(A) 5 (B) 6 (C) 7 (D) 8 (E) 9
- Minneapolis-St. Paul International Airport is 8 miles southwest of downtown St. Paul and 10 miles southeast of downtown Minneapolis. Which of the following is closest to the number of miles between downtown St. Paul and downtown Minneapolis?
(A) 13 (B) 14 (C) 15 (D) 16 (E) 17
- A square has sides of length 10, and a circle centered at one of its vertices has radius 10. What is the area of the union of the regions enclosed by the square and the circle?
(A) $200 + 25\pi$ (B) $100 + 75\pi$ (C) $75 + 100\pi$ (D) $100 + 100\pi$ (E) $100 + 125\pi$
- A grocer makes a display of cans in which the top row has one can and each lower row has two more cans than the row above it. If the display contains 100 cans, how many rows does it contain?
(A) 5 (B) 8 (C) 9 (D) 10 (E) 11
- The point $(-3, 2)$ is rotated 90° clockwise around the origin to point B . Point B is then reflected in the line $y = x$ to point C . What are the coordinates of C ?
(A) $(-3, -2)$ (B) $(-2, -3)$ (C) $(2, -3)$ (D) $(2, 3)$ (E) $(3, 2)$

10. An *annulus* is the region between two concentric circles. The concentric circles in the figure have radii b and c , with $b > c$. Let \overline{OX} be a radius of the larger circle, let \overline{XZ} be tangent to the smaller circle at Z , and let \overline{OY} be the radius of the larger circle that contains Z . Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?

- (A) πa^2 (B) πb^2 (C) πc^2 (D) πd^2 (E) πe^2



11. All the students in an algebra class took a 100-point test. Five students scored 100, each student scored at least 60, and the mean score was 76. What is the smallest possible number of students in the class?

- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

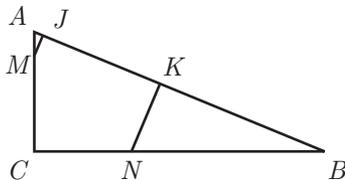
12. In the sequence 2001, 2002, 2003, \dots , each term after the third is found by subtracting the previous term from the sum of the two terms that precede that term. For example, the fourth term is $2001 + 2002 - 2003 = 2000$. What is the 2004th term in this sequence?

- (A) -2004 (B) -2 (C) 0 (D) 4003 (E) 6007

13. If $f(x) = ax + b$ and $f^{-1}(x) = bx + a$ with a and b real, what is the value of $a + b$?

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

14. In $\triangle ABC$, $AB = 13$, $AC = 5$ and $BC = 12$. Points M and N lie on \overline{AC} and \overline{BC} , respectively, with $CM = CN = 4$. Points J and K are on \overline{AB} so that \overline{MJ} and \overline{NK} are perpendicular to \overline{AB} . What is the area of pentagon $CMJKN$?



- (A) 15 (B) $\frac{81}{5}$ (C) $\frac{205}{12}$ (D) $\frac{240}{13}$ (E) 20
15. The two digits in Jack's age are the same as the digits in Bill's age, but in reverse order. In five years Jack will be twice as old as Bill will be then. What is the difference in their current ages?
- (A) 9 (B) 18 (C) 27 (D) 36 (E) 45
16. A function f is defined by $f(z) = i\bar{z}$, where $i = \sqrt{-1}$ and \bar{z} is the complex conjugate of z . How many values of z satisfy both $|z| = 5$ and $f(z) = z$?
- (A) 0 (B) 1 (C) 2 (D) 4 (E) 8
17. For some real numbers a and b , the equation

$$8x^3 + 4ax^2 + 2bx + a = 0$$

has three distinct positive roots. If the sum of the base-2 logarithms of the roots is 5, what is the value of a ?

- (A) -256 (B) -64 (C) -8 (D) 64 (E) 256
18. Points A and B are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of \overline{AB} . What is the length of \overline{AB} ?
- (A) $2\sqrt{5}$ (B) $5 + \frac{\sqrt{2}}{2}$ (C) $5 + \sqrt{2}$ (D) 7 (E) $5\sqrt{2}$
19. A truncated cone has horizontal bases with radii 18 and 2. A sphere is tangent to the top, bottom, and lateral surface of the truncated cone. What is the radius of the sphere?
- (A) 6 (B) $4\sqrt{5}$ (C) 9 (D) 10 (E) $6\sqrt{3}$
20. Each face of a cube is painted either red or blue, each with probability $1/2$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?
- (A) $\frac{1}{4}$ (B) $\frac{5}{16}$ (C) $\frac{3}{8}$ (D) $\frac{7}{16}$ (E) $\frac{1}{2}$

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WRITE TO US!

Correspondence about the problems and solutions for this AMC 12 should be addressed to:

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Orders for any of the publications listed below should be addressed to:

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Lincoln, NE 68501-1606
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2004 AIME

The AIME will be held on Tuesday, March 23, 2004 with the alternate on April 6, 2004. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 1% of the AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) in late Spring. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

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- AMC 10 2000-2004/(AHSME) AMC 12 1989-2004, \$1 per exam copy.
- AIME 1989-2004, \$2 per copy per year (2004 available after April).
- USA and International Math Olympiads, 1989-1999, \$5 per copy per year, 2000-\$14, 2001-\$17
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2004

AMC 12 - Contest B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 25, 2004

****Administration On An Earlier Date Will Disqualify
Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHER'S MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 25.** Nothing is needed from inside this package until February 25.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC no later than 24 hours following the examination.
4. Please Note: All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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