Wednesday, FEBRUARY 26, 2003

Contest B

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions
Presented by the Akamai Foundation

AMC 12

54th Annual American Mathematics Contest 12

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 21st annual American Invitational Mathematics Examination (AIME) on Tuesday, March 25, 2003 or Tuesday, April 8, 2003. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. Which of the following is the same as

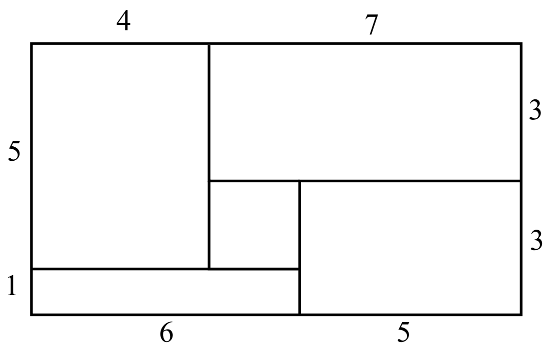
$$\frac{2-4+6-8+10-12+14}{3-6+9-12+15-18+21}?$$

(A) -1 **(B)** $-\frac{2}{3}$ **(C)** $\frac{2}{3}$ **(D)** 1 **(E)** $\frac{14}{3}$

2. Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs \$1 more than a pink pill, and Al's pills cost a total of \$546 for the two weeks. How much does one green pill cost?

(A) \$7 **(B)** \$14 **(C)** \$19 **(D)** \$20 **(E)** \$39

3. Rose fills each of the rectangular regions of her rectangular flower bed with a different type of flower. The lengths, in feet, of the rectangular regions in her flower bed are as shown in the figure. She plants one flower per square foot in each region. Asters cost \$1 each, begonias \$1.50 each, cannas \$2 each, dahlias \$2.50 each, and Easter lilies \$3 each. What is the least possible cost, in dollars, for her garden?

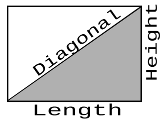


(A) 108 **(B)** 115 **(C)** 132 **(D)** 144 **(E)** 156

4. Moe uses a mower to cut his rectangular 90-foot by 150-foot lawn. The swath he cuts is 28 inches wide, but he overlaps each cut by 4 inches to make sure that no grass is missed. He walks at the rate of 5000 feet per hour while pushing the mower. Which of the following is closest to the number of hours it will take Moe to mow his lawn?

(A) 0.75 **(B)** 0.8 **(C)** 1.35 **(D)** 1.5 **(E)** 3

5. Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is $4 : 3$. The horizontal length of a "27-inch" television screen is closest, in inches, to which of the following?



(A) 20 **(B)** 20.5 **(C)** 21 **(D)** 21.5 **(E)** 22

6. The second and fourth terms of a geometric sequence are 2 and 6. Which of the following is a possible first term?

(A) $-\sqrt{3}$ **(B)** $-\dfrac{2\sqrt{3}}{3}$ **(C)** $-\dfrac{\sqrt{3}}{3}$ **(D)** $\sqrt{3}$ **(E)** 3

7. Penniless Pete's piggy bank has no pennies in it, but it has 100 coins, all nickels, dimes, and quarters, whose total value is \$8.35. It does not necessarily contain coins of all three types. What is the difference between the largest and smallest number of dimes that could be in the bank?

(A) 0 **(B)** 13 **(C)** 37 **(D)** 64 **(E)** 83

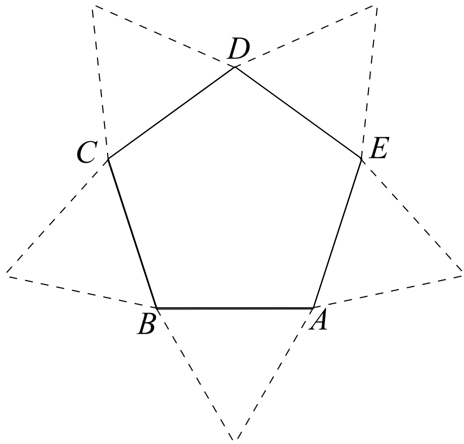
8. Let $\clubsuit(x)$ denote the sum of the digits of the positive integer x. For example, $\clubsuit(8) = 8$ and $\clubsuit(123) = 1 + 2 + 3 = 6$. For how many two-digit values of x is $\clubsuit(\clubsuit(x)) = 3$?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 9 **(E)** 10

9. Let f be a linear function for which $f(6) - f(2) = 12$. What is $f(12) - f(2)$?

(A) 12 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

10. Several figures can be made by attaching two equilateral triangles to the regular pentagon $ABCDE$ in two of the five positions shown. How many non-congruent figures can be constructed in this way?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

11. Cassandra sets her watch to the correct time at noon. At the actual time of 1:00 PM, she notices that her watch reads 12:57 and 36 seconds. Assuming that her watch loses time at a constant rate, what will be the actual time when her watch first reads 10:00 PM?

(A) 10:22 PM and 24 seconds **(B)** 10:24 PM **(C)** 10:25 PM
(D) 10:27 PM **(E)** 10:30 PM

12. What is the largest integer that is a divisor of

$$(n+1)(n+3)(n+5)(n+7)(n+9)$$

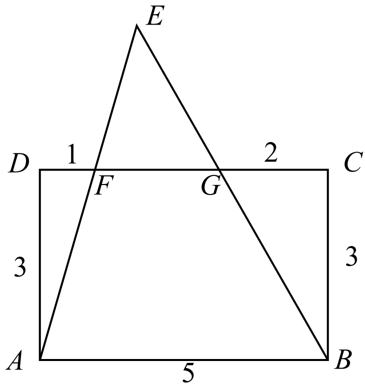
for all positive even integers n?

(A) 3 **(B)** 5 **(C)** 11 **(D)** 15 **(E)** 165

13. An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius?

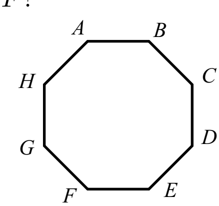
(A) $2:1$ **(B)** $3:1$ **(C)** $4:1$ **(D)** $16:3$ **(E)** $6:1$

14. In rectangle $ABCD$, $AB = 5$ and $BC = 3$. Points F and G are on $\overline{CD}$ so that $DF = 1$ and $GC = 2$. Lines AF and BG intersect at E. Find the area of $\triangle AEB$.



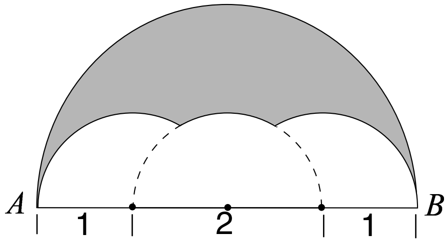
(A) 10 **(B)** $\frac{21}{2}$ **(C)** 12 **(D)** $\frac{25}{2}$ **(E)** 15

15. A regular octagon $ABCDEFGH$ has an area of one square unit. What is the area of the rectangle $ABEF$?



(A) $1 - \frac{\sqrt{2}}{2}$ **(B)** $\frac{\sqrt{2}}{4}$ **(C)** $\sqrt{2} - 1$ **(D)** $\frac{1}{2}$ **(E)** $\frac{1+\sqrt{2}}{4}$

16. Three semicircles of radius 1 are constructed on diameter $\overline{AB}$ of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal length, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?



(A) $\pi - \sqrt{3}$ **(B)** $\pi - \sqrt{2}$ **(C)** $\dfrac{\pi + \sqrt{2}}{2}$ **(D)** $\dfrac{\pi + \sqrt{3}}{2}$

(E) $\dfrac{7}{6}\pi - \dfrac{\sqrt{3}}{2}$

17. If $\log(xy^3) = 1$ and $\log(x^2y) = 1$, what is $\log(xy)$?

(A) $-\dfrac{1}{2}$ **(B)** 0 **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{3}{5}$ **(E)** 1

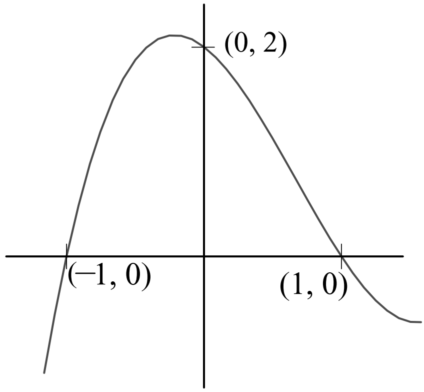
18. Let x and y be positive integers such that $7x^5 = 11y^{13}$. The minimum possible value of x has a prime factorization a^cb^d. What is $a + b + c + d$?

(A) 30 **(B)** 31 **(C)** 32 **(D)** 33 **(E)** 34

19. Let S be the set of permutations of the sequence $1, 2, 3, 4, 5$ for which the first term is not 1. A permutation is chosen randomly from S. The probability that the second term is 2, in lowest terms, is a/b. What is $a + b$?

(A) 5 **(B)** 6 **(C)** 11 **(D)** 16 **(E)** 19

20. Part of the graph of $f(x) = ax^3 + bx^2 + cx + d$ is shown. What is b?

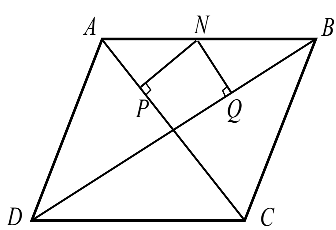


(A) -4 **(B)** -2 **(C)** 0 **(D)** 2 **(E)** 4

21. An object moves 8 cm in a straight line from A to B, turns at an angle α, measured in radians and chosen at random from the interval $(0, \pi)$, and moves 5 cm in a straight line to C. What is the probability that $AC < 7$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

22. Let $ABCD$ be a rhombus with $AC = 16$ and $BD = 30$. Let N be a point on $\overline{AB}$, and let P and Q be the feet of the perpendiculars from N to $\overline{AC}$ and $\overline{BD}$, respectively. Which of the following is closest to the minimum possible value of PQ?



(A) 6.5 **(B)** 6.75 **(C)** 7 **(D)** 7.25 **(E)** 7.5

23. The number of x-intercepts on the graph of $y = \sin(1/x)$ in the interval $(0.0001, 0.001)$ is closest to

(A) 2900 **(B)** 3000 **(C)** 3100 **(D)** 3200 **(E)** 3300

24. Positive integers a, b, and c are chosen so that $a < b < c$, and the system of equations

$$2x + y = 2003 \quad \text{and} \quad y = |x - a| + |x - b| + |x - c|$$

has exactly one solution. What is the minimum value of c?

(A) 668 **(B)** 669 **(C)** 1002 **(D)** 2003 **(E)** 2004

25. Three points are chosen randomly and independently on a circle. What is the probability that all three pairwise distances between the points are less than the radius of the circle?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{24}$ **(C)** $\frac{1}{18}$ **(D)** $\frac{1}{12}$ **(E)** $\frac{1}{9}$

WRITE TO US!

*Correspondence about the problems and solutions for this **AMC 12** should be addressed to:*

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Orders for any of the publications listed below should be addressed to:

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University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: titu@amc.unl.edu;

2003 AIME

The AIME will be held on Tuesday, March 25, 2003 with the alternate on April 8,2003. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of the AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) in late Spring. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

MINIMUM ORDER: $10 (before shipping/handling fee), *PAYMENT IN US FUNDS ONLY made payable to the* American Mathematics Competitions or VISA/MASTERCARD/AMERICAN EXPRESS accepted. Include card number, expiration date, cardholder name and address. U.S.A. and Canadian orders must be prepaid and will be shipped Priority Mail, UPS or Air Mail.

INTERNATIONAL ORDERS: Do **NOT** prepay. An invoice will be sent to you.

Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2002.

- AMC 10 2000-2002/AHSME (AMC 12) 1989-2002, $1 per copy per year.
- AIME 1989-2002, $2 per copy per year.
- USA and International Math Olympiads, 1989-1999, $5 per copy per year, 2000, $14.00 per year.
- National Summary of Results and Awards, 1989-2002, $10 per copy per year.
- Problem Book I, AHSMEs 1950-60, Problem Book II, AHSMEs 1961-65, $10/ea
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2003

AMC 12 - Contest B

DO NOT OPEN UNTIL

Wednesday, FEBRUARY 26, 2003

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 26.** Nothing is needed from inside this package until February 26.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. <u>Please Note:</u> All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
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