Tuesday, FEBRUARY 11, 2003

Contest A

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions
Presented by the Akamai Foundation

AMC 12

54th Annual American Mathematics Contest 12

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 12 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 100 or above or finish in the top 5% on this AMC 12 will be invited to take the 21st annual American Invitational Mathematics Examination (AIME) on Tuesday, March 25, 2003 or Tuesday, April 8, 2003. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. What is the difference between the sum of the first 2003 even counting numbers and the sum of the first 2003 odd counting numbers?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 2003 **(E)** 4006

2. Members of the Rockham Soccer League buy socks and T–shirts. Socks cost \$4 per pair and each T–shirt costs \$5 more than a pair of socks. Each member needs one pair of socks and a shirt for home games and another pair of socks and a shirt for away games. If the total cost is \$2366, how many members are in the League?

   (A) 77 **(B)** 91 **(C)** 143 **(D)** 182 **(E)** 286

3. A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of this box. What percent of the original volume is removed?

   (A) 4.5 **(B)** 9 **(C)** 12 **(D)** 18 **(E)** 24

4. It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to home along the same route. What is her average speed, in km/hr, for the round trip?

   (A) 3 **(B)** 3.125 **(C)** 3.5 **(D)** 4 **(E)** 4.5

5. The sum of the two 5-digit numbers $AMC10$ and $AMC12$ is 123422. What is $A + M + C$?

   (A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

6. Define $x \heartsuit y$ to be $|x - y|$ for all real numbers x and y. Which of the following statements is **not** true?

   (A) $x \heartsuit y = y \heartsuit x$ for all x and y

   (B) $2(x \heartsuit y) = (2x) \heartsuit (2y)$ for all x and y **(C)** $x \heartsuit 0 = x$ for all x

   (D) $x \heartsuit x = 0$ for all x **(E)** $x \heartsuit y > 0$ if $x \neq y$

7. How many non-congruent triangles with perimeter 7 have integer side lengths?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. What is the probability that a randomly drawn positive factor of 60 is less than 7?

   (A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

9. A set S of points in the xy-plane is symmetric about the origin, both coordinate axes, and the line $y = x$. If $(2, 3)$ is in S, what is the smallest number of points in S?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8 **(E)** 16

10. Al, Bert, and Carl are the winners of a school drawing for a pile of Halloween candy, which they are to divide in a ratio of $3 : 2 : 1$, respectively. Due to some confusion they come at different times to claim their prizes, and each assumes he is the first to arrive. If each takes what he believes to be his correct share of candy, what fraction of the candy goes unclaimed?

(A) $\frac{1}{18}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{5}{18}$ **(E)** $\frac{5}{12}$

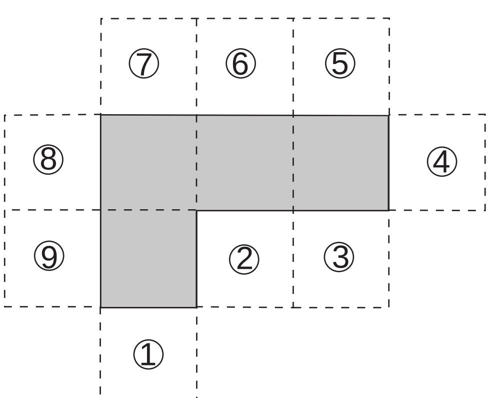
11. A square and an equilateral triangle have the same perimeter. Let A be the area of the circle circumscribed about the square and B be the area of the circle circumscribed about the triangle. Find A/B.

(A) $\frac{9}{16}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{27}{32}$ **(D)** $\frac{3\sqrt{6}}{8}$ **(E)** 1

12. Sally has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card divides evenly into the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

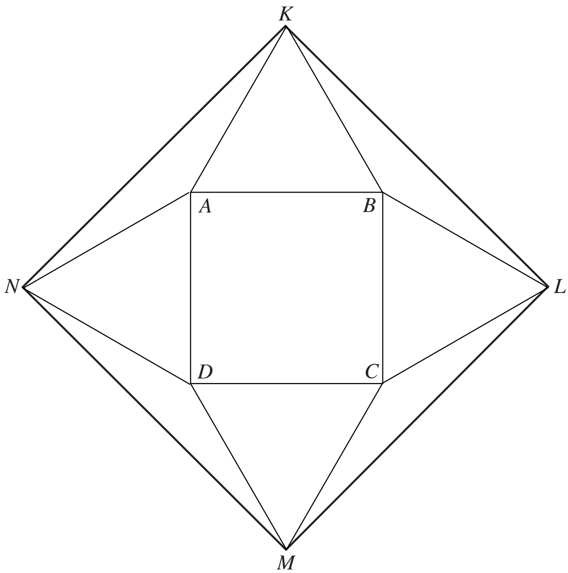
(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

13. The polygon enclosed by the solid lines in the figure consists of 4 congruent squares joined edge-to-edge. One more congruent square is attached to an edge at one of the nine positions indicated. How many of the nine resulting polygons can be folded to form a cube with one face missing?



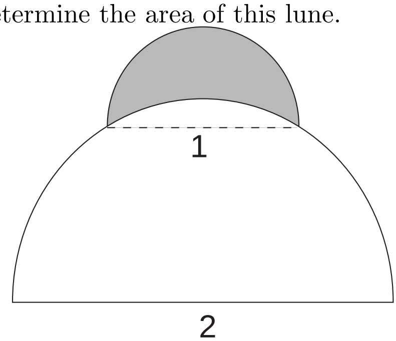
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

14. Points K, L, M, and N lie in the plane of the square $ABCD$ so that AKB, BLC, CMD, and DNA are equilateral triangles. If $ABCD$ has an area of 16, find the area of $KLMN$.



(A) 32 **(B)** $16+16\sqrt{3}$ **(C)** 48 **(D)** $32+16\sqrt{3}$ **(E)** 64

15. A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.



(A) $\frac{1}{6}\pi-\frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{4}-\frac{1}{12}\pi$ **(C)** $\frac{\sqrt{3}}{4}-\frac{1}{24}\pi$ **(D)** $\frac{\sqrt{3}}{4}+\frac{1}{24}\pi$

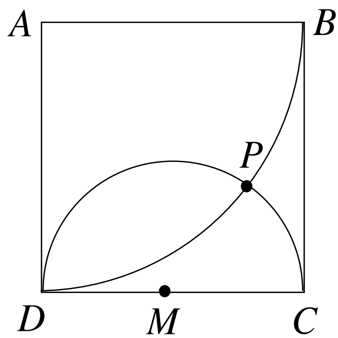
(E) $\frac{\sqrt{3}}{4}+\frac{1}{12}\pi$

16. A point P is chosen at random in the interior of equilateral triangle ABC. What is the probability that $\triangle ABP$ has a greater area than each of $\triangle ACP$ and $\triangle BCP$?

(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

17. Square $ABCD$ has sides of length 4, and M is the midpoint of $\overline{CD}$. A circle with radius 2 and center M intersects a circle with radius 4 and center A at points P and D. What is the distance from P to $\overline{AD}$?

(A) 3 **(B)** $\frac{16}{5}$ **(C)** $\frac{13}{4}$ **(D)** $2\sqrt{3}$ **(E)** $\frac{7}{2}$



18. Let n be a 5-digit number, and let q and r be the quotient and remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?

(A) 8180 **(B)** 8181 **(C)** 8182 **(D)** 9000 **(E)** 9090

19. A parabola with equation $y = ax^2 + bx + c$ is reflected about the x-axis. The parabola and its reflection are translated horizontally five units in opposite directions to become the graphs of $y = f(x)$ and $y = g(x)$, respectively. Which of the following describes the graph of $y = (f + g)(x)$?

(A) a parabola tangent to the x-axis

(B) a parabola not tangent to the x-axis **(C)** a horizontal line

(D) a non-horizontal line **(E)** the graph of a cubic function

20. How many 15-letter arrangements of 5 A's, 5 B's, and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters, and no C's in the last 5 letters?

(A) $\sum_{k=0}^{5} \binom{5}{k}^3$ **(B)** $3^5 \cdot 2^5$ **(C)** 2^{15} **(D)** $\frac{15!}{(5!)^3}$ **(E)** 3^{15}

21. The graph of the polynomial

$$P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$$

has five distinct x-intercepts, one of which is at $(0,0)$. Which of the following coefficients cannot be zero?

(A) a **(B)** b **(C)** c **(D)** d **(E)** e

22. Objects A and B move simultaneously in the coordinate plane via a sequence of steps, each of length one. Object A starts at $(0,0)$ and each of its steps is either right or up, both equally likely. Object B starts at $(5,7)$ and each of its steps is either left or down, both equally likely. Which of the following is closest to the probability that the objects meet?

(A) 0.10 **(B)** 0.15 **(C)** 0.20 **(D)** 0.25 **(E)** 0.30

23. How many perfect squares are divisors of the product $1! \cdot 2! \cdot 3! \cdots 9!$?

(A) 504 **(B)** 672 **(C)** 864 **(D)** 936 **(E)** 1008

24. If $a \geq b > 1$, what is the largest possible value of $\log_a(a/b) + \log_b(b/a)$?

(A) -2 **(B)** 0 **(C)** 2 **(D)** 3 **(E)** 4

25. Let $f(x) = \sqrt{ax^2 + bx}$. For how many real values of a is there at least one positive value of b for which the domain of f and the range of f are the same set?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** infinitely many

WRITE TO US!

Correspondence about the problems and solutions for this ***AMC 12*** *should be addressed to:*

Prof. David Wells, Department of Mathematics
Penn State University, New Kensington, PA 15068
Phone: 724/334-6749; Fax: 724/334-6110; email: dmw8@psu.edu

Orders for any of the publications listed below should be addressed to:

Titu Andreescu, Director
American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: tandreescu2@unl.edu;

2003 AIME

The AIME will be held on Tuesday, March 25, 2003 with the alternate on April 8,2003. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of the AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) in late Spring. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

MINIMUM ORDER: $10 (before shipping/handling fee), *PAYMENT IN US FUNDS ONLY made payable to the* American Mathematics Competitions or VISA/MASTERCARD/AMERICAN EXPRESS accepted. Include card number, expiration date, cardholder name and address. U.S.A. and Canadian orders must be prepaid and will be shipped Priority Mail, UPS or Air Mail.
INTERNATIONAL ORDERS: Do **NOT** prepay. An invoice will be sent to you.

Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2003.

- AMC 10 2000-2003/AHSME (AMC 12) 1989-2003, $1 per copy per year.
- AIME 1989-2003, $2 per copy per year (2003 available after April).
- USA and International Math Olympiads, 1989-1999, $5 per copy per year, 2000-$14, 2001-$17
- National Summary of Results and Awards, 1989-2003, $10 per copy per year.
- Problem Book I, AHSMEs 1950-60, Problem Book II, AHSMEs 1961-65, $10/ea
- Problem Book III, AHSMEs 1966-72, Problem Book IV, AHSMEs 1973-82, $13/ea
- Problem Book V, AHSMEs and AIMEs 1983-88, $30/ea
- Problem Book VI, AHSMEs 1989-1994, $24/ea
- USA Mathematical Olympiad Book 1972-86, $18/ea
- International Mathematical Olympiad Book II, 1978-85, $20/ea
- World Olympiad Problems/Solutions 1995-96, 1996-97, 1997-98, $15/ea
- Mathematical Olympiads Problems & Solutions from around the World 1998-1999, 1999-2000 $25/ea
- The Arbelos, Volumes I-V, and a Special Geometry Issue, $8/ea

Shipping & Handling charges for Publication Orders:

Order Total	Add:	Order Total	Add:
\$ 10.00 -- \$ 30.00	\$ 5	\$ 40.01 -- \$ 50.00	\$ 9
\$ 30.01 -- \$ 40.00	\$ 7	\$ 50.01 -- \$ 75.00	\$12
\$ 75.01 -- up	\$15		

2003

AMC 12 - Contest A

DO NOT OPEN UNTIL TUESDAY, FEBRUARY 11, 2003

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 11.** Nothing is needed from inside this package until February 11.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. <u>Please Note:</u> All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.*

Sponsored by
The MATHEMATICAL ASSOCIATION OF AMERICA
The Akamai Foundation
University of Nebraska – Lincoln
Contributors
American Statistical Association
Casualty Actuarial Society
Society of Actuaries
National Council of Teachers of Mathematics
American Society of Pension Actuaries
American Mathematical Society
American Mathematical Association of Two Year Colleges
Pi Mu Epsilon
Consortium for Mathematics and its Applications
Mu Alpha Theta
National Association of Mathematicians
Kappa Mu Epsilon
School Science and Mathematics Association
Clay Mathematics Institute
Institute for Operations Research and the Management Sciences
Canada/USA Mathpath & Mathcamp